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Abstract

Evaluating network effects and two-sidedness is critical for merger control in the digital economy. To examine the impact of network effects on the welfare properties of mergers, this study analyzes a model of multiproduct-firm oligopoly with firm-level direct and indirect network externalities using an aggregative-games approach. The analysis shows that network externalities increase both the consumer benefits of mergers through network expansion and the cost of accompanying market power. The former justifies mergers involving small firms, but the latter makes mergers between dominant firms more likely to hurt consumers. In two-sided markets, the effect of mergers on consumer surplus depends on merging parties' pre-merger price structures. In particular, when a consumer group is subsidized through two-sided pricing by merging parties, such consumers are likely to benefit from mergers. These results provide theoretical guidance on merger policy toward platforms.

Keywords: Aggregative games; merger policy; network externalities; innovation; two-sided markets

JEL Codes: L13; L41

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1. Introduction

In the digital economy, several markets are characterized by network externalities and multi-sidedness, where platform operators generate direct and indirect network externalities by facilitating the interaction among the participants. While network effects benefit consumers in many cases, the positive-feedback effect of network externalities often leads consumers to focus on a few dominant platforms. This "winner-takes-all" feature makes competition authorities raise concern about the potential harm of persistent dominance of particular firms.[1]

Furthermore, dominant firms in the tech industry often expand their networks through mergers and acquisitions. Such acquisitions include many small and several large ones, such as Google/DoubleClick, Microsoft/LinkedIn, Apple/Shazam, and Facebook/WhatsApp, which were subject to scrutiny by competition authorities. The common issue in these merger reviews is the evaluation of the impact of network effects and two-sidedness on the desirability of mergers. While it is recognized that merger policy must consider network effects and multi-sidedness of platform businesses (Evans and Noel, 2008; Ocello and Sjödin, 2018), the theoretical guidance on these issues is still scarce. This lack of guidance may hurt the society through incorrect decisions on the prospective mergers between platforms.

The aim of this study is to provide a theoretical guidance on merger policy in industries characterized by network externalities. To this end, I develop a model of multiproduct-firm oligopoly with direct and indirect network externalities that allow for an arbitrary number of heterogeneous firms. In particular, this study introduces firm-level network externalities to CES and multinomial-logit demand systems and applies an aggregative-games approach (Nocke and Schutz, 2018b) to characterize the equilibrium in the pricing game among firms with different product portfolios. In this framework, as in Nocke and Schutz (2018b), type-aggregation obtains: all the relevant information for the pricing of each firm is summarized in some sufficient statistics, named as the firm's "type," the dimension of which equals the number of sides of the markets. Therefore, any merger can be formalized as a change in the profile of types, which enables the characterization of the competitive effects of mergers by analyzing the impact of changes in types. In particular, by defining a merger-specific synergy as an increase in types following a merger, one can characterize the synergies required for mergers to improve consumer surplus. Characterizing such synergies gives conditions under which a merger is likely to benefit or harm consumers.

Using the framework described earlier, this study offers three sets of merger analyses: mergers in the presence of direct network externalities, acquisitions of innovative entrants by incumbents, and merges in two-sided markets. In all the three analyses, the presence of direct and indirect network externalities changes the market-power effects of mergers through demand-side scale economies and changes in subsidization incentives. The directions and magnitudes of such changes depend on the pre-merger sizes of the merging parties relative to the markets.

From the perspective that the objective of competition authorities can be approximated by

[1]For example, Australian Competition and Consumer Commission released "Digital Platforms Inquiry" that tries to identify the market powers of digital platforms (`https://www.accc.gov.au/focus-areas/inquiries/digital-platforms-inquiry`).

consumer surplus (Whinston, 2007), I first examine the impact of mergers on consumer surplus in the presence of direct network externalities. To this end, I explore how the size of the synergy required for a merger to improve consumer surplus varies based on the sizes of the merging parties and the magnitude of the network externalities. Network externalities have two countervailing effects on the welfare properties of mergers. While consumers directly benefit from the network externalities arising from mergers because of the expansion of the merged entity's networks, network externalities might also increase the market power of the merged entity, which accompanies the higher markups. The overall impact of network externalities on the competitive effects of mergers depends on the relative magnitudes of these two effects.

Several cases in which one effect dominates the other are identified. For example, when the merging parties are small or when the firms in an industry are symmetric, the synergy required for a merger to improve consumer surplus decreases with the magnitude of network externalities or even become negative. This is because the benefit from network expansion dominates the cost of incremental market power. Accordingly, the consumer surplus is improved by the presence of network externalities in mergers involving non-dominant firms. However, when the merging parties are dominant in the industry, the presence of network externalities increases the synergy required for a merger to improve the consumer surplus. Thus, the merger is likely to hurt consumers. This follows from the fact that when the merging parties are sufficiently large compared with the industry, the presence of network effects enables the merged entity who is equipped with a huge customer base to easily attract consumers without lowering the prices. In this case, the merged entity can easily exert its market power to increase markups, which in turn decreases consumer welfare. In total, network externalities can serve as a justification for mergers involving small firms or mergers in an industry with symmetric firms. However, as the size of merging parties grows relative to the industry, the presence of network externalities requires the more intense scrutiny of merger reviews.

To contribute to the recent discussion on “killer acquisitions" (Cunningham, Ederer and Ma, 2018), I extend the model to incorporate the case where an incumbent tries to acquire an innovative potential entrant. This extension shows that when the size of a potential innovation is small, the incumbent has greater incentive to continue the project than the entrant does, while the opposite may hold when the size of a potential innovation is large. When the size of the innovation is small, the entrant suffers from an inability to expand its network, while the incumbent can leverage its installed base to effectively sell the new product. This demand-side scale economy makes the incumbent more willing to innovate, which is in sharp contrast with the replacement effect argument that the incumbent has smaller incentive to innovate because the new product just “replaces" the existing product. This result suggests that if the authorities adopt a too aggressive merger policy and ban acquisitions of innovative entrants such as tech-startups, investors might stop funding some innovative projects at all. This observation is consistent with the remark put by Bruce Hoffman at the Federal Trade Commission that “[t]o the extent exit strategies for startups involve acquisitions, if such acquisition opportunities are constrained, the capital available for startups may fall. That, in turn, could result in fewer startups."[2] Given the fact that a large fraction of exit strategies adopted by startups are M&A, this is a serious concern.[3]

Finally, I analyze mergers in the presence of indirect network externalities, with a focus on two-sided markets. In the presence of indirect network externalities, firms typically engage in two-sided pricing: they subsidize consumers on one side of the market by charging lower prices, while collecting revenues from consumers on the other side of the market by charging higher prices at the same time. Such subsidization incentives are interrelated with the size of the firm on each side of the market (Weyl, 2010). Mergers thus affect not only the market power but also the subsidization incentives and the price structure among multiple sides of markets, which leads to a change in the division of surplus between different groups of consumers.

Focusing on a relatively simple case where only consumers on one side gain from indirect network externalities, I examine the relation between the pre-merger sizes of the merging parties and the consumer-surplus effects of mergers. I show that the synergies on the two sides of markets required for any merger to leave consumer surplus on both the sides unchanged, named as CS-neutral synergies, can be negative depending on the pre-merger sizes and the price structures of the merging parties. In particular, CS-neutral synergy on the side that benefits from indirect network externalities (subsidizing segment) is negative when the sizes of merging parties are small. This result is analogous to the effect of direct network externalities: when the merging parties are small, the benefit from a network expansion outweighs the cost of an increase in market power. In addition, CS-neutral synergy on the side that generates indirect network externalities (subsidized segment) is negative when the merging parties set negative markups on that side, which occurs when the merging parties are relatively large on the subsidizing segment. This is because the increase in the size on the subsidizing segment following the merger increases the incentive to subsidize the consumers on the subsidized segment. Therefore a merger that involves firms with a large pre-merger share on the subsidizing segment is more likely to benefit consumers on the subsidized segment.

The results of this study provide theoretical guidance on merger policy toward platforms. In the presence of network externalities, mergers between non-dominant firms are more likely to benefit consumers, while mergers that lead to extreme concentration are more likely to hurt consumers. Killer acquisitions are of less concern when the sizes of potential innovations are small. In two-sided markets, both pre-merger price structures and sizes provide relevant information on the effects of mergers on consumers.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents a model of multiproduct-firm oligopoly with direct network externalities. Sections 4 then analyzes the impact of mergers on consumer surplus in the presence of network

[2]"Competition Policy and the Tech Industry – What's at stake?" Available at (https://www.ftc.gov/system/files/documents/public_statements/1375444/ccia_speech_final_april30.pdf). For an opposite argument that loose merger policy may discourage the investment in startups, see Kamepalli, Rajan and Zingales (2020).

[3]In the annual report at 2016, CB Insights notes that there were 3,358 total tech exits in the year, and that 3,260 of them were M&A exits. See CB Insights, "2016 Global Tech Exits Report," (https://www.cbinsights.com/research/report/tech-exits-2016/).

externalities. Section 5 and 6 analyze acquisitions of innovative entrants and mergers in two-sided markets. Section 7 discusses several issues abstracted in the main analysis. Section 8 concludes.

2. Related Literature

The present study is related to four strands of literature: network externalities, two-sided markets, innovation, and horizontal mergers.

The first strand of literature analyzes the impact of network externalities on the consumer choice of technologies (Farrel and Saloner, 1986) and firms' pricing in static and dynamic environments (Katz and Shapiro, 1985; Cabral, 2011). To the best of my knowledge, there has been no systematic theoretical study on the relation between the network externalities and horizontal mergers, because of the lack of the tractable framework to analyze the merger in flexible environments. By introducing the network externalities to the framework of multiproduct-firm oligopoly proposed by Nocke and Schutz (2018b), this study provides a systematic theoretical analysis of the impact of network externalities on the welfare properties of mergers.

The second strand of literature studies competition in two-sided markets, with a focus on the determinants of price structures in the presence of indirect network externalities (Rochet and Tirole, 2003, 2006; Armstrong, 2006; Weyl, 2010). Several recent studies theoretically analyze the impact of mergers between two-sided platforms on consumer welfare under single-homing (Baranes, Cortade and Cosnita-Langlais, 2014; Tan and Zhou, 2017; Correia-da-Silva, Jullien, Lefouili and Pinho, 2019), competitive-bottleneck (Anderson and Peitz, 2020), and multi-homing situations (Anderson, Foros and Kind, 2019). This study uses a model with single-homing consumers (Baranes et al., 2014; Tan and Zhou, 2017; Correia-da-Silva et al., 2019). Including the present study, this strand of literature faces a trade-off between analyzing the general market environments and providing clear welfare implications. Using demand systems that enables to adopt an aggregative-games approach developed by Nocke and Schutz (2018b), this study allows for an arbitrary number of heterogeneous firms and obtains a clear characterization of the consumer-surplus effects of mergers with respect to pre-merger sizes and price structures of the merging parties.

The third strand of literature studies the impact of mergers on innovation. (Motta and Tarantino, 2017; Federico, Langus and Valletti, 2018; Cunningham et al., 2018).[4] Cunningham et al. (2018) use a simple model of the acquisition of innovative entrant by an incumbent and show that due to the replacement effect, the incumbent always has a smaller incentive to innovate than the entrant does. They empirically confirmed this prediction by examining the progress of R&D projects in the pharmaceutical industry accompanying the acquisitions. The present study adopts the same timeline as Cunningham et al. (2018) do and shows that, in the presence of network externalities, incumbents may have greater incentives to innovate than the entrant does due to the demand-side scale economies. Thus, the presence of network externalities enriches the predictions made by the existing research.

[4] See Jullien and Lefouili (2018) for more comprehensive review.

The last strand of literature studies the welfare properties of horizontal mergers (Williamson, 1968; Farrell and Shapiro, 1990; Nocke and Whinston, 2010, 2013; Nocke and Schutz, 2018b,a). The framework of this study is based on Nocke and Schutz (2018b,a). They show that with a class of demand systems that satisfy IIA properties among the sets of products offered by firms, price competition in multiproduct-firm oligopoly can be expressed as an aggregative game, which largely simplifies the characterization of equilibria. Furthermore, they also show that when the demand system is given by nested multinomial-logit or nested CES demand, all the relevant information for a firm's optimal strategy can be summarized by a scalar-valued "type" and the industry-level aggregator, which enables conducting a tractable merger analysis. This study extends the framework proposed by Nocke and Schutz (2018a) to incorporate firm-level direct and indirect network externalities while simultaneously preserving the aggregative feature of consumer demand. Technically, this study extends their "type-aggregation property" to two-sided markets, where all the relevant information for an optimal strategy of each firm can be summarized in the type that has the dimension corresponding with the number of sides of the markets. This enables the study of the impacts of direct and indirect network externalities on the welfare properties of horizontal mergers by conducting a merger analysis in a similar way as Nocke and Schutz (2018a). Therefore, this study contributes to the existing literature by examining how the results in the standard settings are preserved or altered by the introduction of direct and indirect network externalities. For example, while mergers without synergies hurt the consumer surplus in Nocke and Schutz (2018a)'s environment, such mergers can improve the consumer surplus in the presence of direct or indirect network externalities.

Finally, I mention several studies that provide tools to quantify the effects of horizontal mergers in two-sided markets. Affeldt, Filistrucchi and Klein (2013) propose a simple measure of upward-pricing pressure modified to incorporate the two-sidedness of markets. While the concept of upward-pricing pressure provides insights to policymakers, the presence of network effects and two-sidedness complicates the evaluation of the consumer-surplus effects of mergers solely based on the price levels. In this regard, the present study provides a clear characterization of the consumer-surplus effects of mergers. Jeziorski (2014) provides a merger analysis using a merger simulation based on an estimated structural model. This approach has an advantage in that, once given appropriate data, it can incorporate rich structures and directly quantify how a specific merger would affect consumer welfare. However, the implication of a specific merger simulation is inevitably case-by-case and cannot be easily generalized to the other environments. In this regard, the theoretical implications provided in this study has an advantage in linking the sizes of the merging parties to the welfare effects of mergers and providing simple intuitions that would apply to a range of applications.

3. Model with Direct Network Externalities

This section presents a model of multiproduct-firm oligopoly with firm-level direct network externalities. All details of derivation are provided in Appendix A, which presents the analysis of a generalized model of multiproduct-firm oligopoly in the presence of both direct and indirect network externalities.

3.1. Consumer demand

Consider an industry with a set $\mathcal{N}$ of imperfectly substitutable products produced by a set of firms $\mathcal{F}$. Each firm $f \in \mathcal{F}$ produces the set $\mathcal{N}_f$ of products, where $\mathcal{N}_f \cap \mathcal{N}_{f'} = \emptyset$ for $f \neq f'$ and $\bigcup_{f \in \mathcal{F}} \mathcal{N}_f = \mathcal{N}$. There is a mass of consumers who derive firm-level network externalities from the purchase of each product $i \in \mathcal{N}_f$, which depends on the number of consumers n_f who purchase the products of firm f. In particular, each consumer $z \in [0, 1]$ yields the indirect utility from the purchase of product $i \in \mathcal{N}_f$ by

$$\log h_i(p_i) + \alpha \log n_f + \varepsilon_{iz}, \tag{1}$$

where $\log h_i(p_i)$ is the stand-alone indirect subutility from product i at price p_i, $\alpha \log n_f$ is the utility from the direct network externalities, and ε_{iz} is an idiosyncratic taste shock that follows an i.i.d. type-I extreme value distribution. Sizes of the direct network externalities depend on the number n_f of consumers who purchase products from firm f and the magnitude of direct network externalities $\alpha \in [0, 1)$. The direct network externalities are based on the number of participants in each firm. Such a form of network externalities can be interpreted as membership externalities (Armstrong, 2006; Weyl, 2010). The assumption that network externalities present at a firm level means that products in a firm's single network have greater compatibility than products in different firms' networks.

I adopt two specific forms of functions h_i. One is *MNL-class* demand specification, where

$$h_i(p_i) = \exp\left(\frac{a_i - p_i}{\lambda}\right),$$

and the other is *CES-class* demand specification where

$$h_i(p_i) = \begin{cases} a_i p_i^{1-\sigma} & \text{if } p_i > 0 \\ +\infty & \text{if } p_i \leq 0 \end{cases}$$

with $\sigma > 1$. In both the specifications, a_i represents the quality of each product. I mean by "MNL-class" and "CES-class" that, if $\alpha = 0$, the demand system obtained from indirect subutilities $h_i(p_i) = \exp\left(\frac{a_i - p_i}{\lambda}\right)$ and $h_i(p_i) = a_i p_i^{1-\sigma}$ corresponds with that of multinomial-logit and CES demand functions, respectively.

Given the profiles of network sizes $(n_f)_{f \in \mathcal{F}}$ and prices $p := (p_i)_{i \in \mathcal{N}}$, consumers choose one product to purchase and the amount of the purchase of the product. This implies that consumers single-home.[5] I assume that there is no outside option so that all consumers purchase some product from the set $\mathcal{N}$.[6] With this specification, the demand system is derived as a rational-expectation equilibrium among consumers. In other words, based on the common expectation over network sizes, consumers choose their own decisions to maximize the utilities, and the realized network sizes are consistent with the original expectation.

The demand system under the rational expectation equilibrium is derived in the following

[5]This assumption is not innocuous when we consider digital markets such as social media. I discuss some implications of multi-homing in Section 7.

[6]In Section 7 discusses the way to relax this assumption.

manner. First, the firm-level and industry-level aggregators are defined as follows:

$$H_f(p_f) = \sum_{i\in\mathcal{N}_f} h_i(p_i),$$

where $p_f := (p_i)_{i\in\mathcal{N}_f}$, and

$$H(p) = \sum_{f\in\mathcal{F}} \left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}.$$

Next, I derive demand for each product conditional on the purchase. Applying Roy's identity, the conditional demand function for product $i \in \mathcal{N}$ conditional on the purchase is given by $-h_i'(p_i)/h_i(p_i)$. I assume that consumers form the correct expectation that all firms have positive network shares. I call the network choice of consumers based on such expectation as an *interior consumption equilibrium*. Applying the Holman and Marley's Theorem, the consumer choice probability s_i of product $i \in \mathcal{N}_f$ given the expectation over network shares $(n_{f'})_{f'\in\mathcal{F}}$ is given by

$$s_i = \frac{h_i(p_i)\left(n_f\right)^\alpha}{\sum_{f'\in\mathcal{F}}\sum_{j\in\mathcal{N}_{f'}} h_j(p_j)\left(n_{f'}\right)^\alpha}. \tag{2}$$

In the interior consumption equilibrium, the network shares are consistent with the consumers' behaviors so that the network share n_f of firm f is given by the sum of the choice probability of products produced by firm f:

$$n_f = \sum_{i\in\mathcal{N}_f} s_i. \tag{3}$$

Note that there are other trivial equilibria in the consumers' network choices where a set of products is expected to be purchased by no consumers. I exclude such equilibria from consideration.[7] From equations (2) and (3), the share of product $i \in \mathcal{N}_f$ in the set of products sold by firm f is given by

$$\frac{s_i}{n_f} = \frac{h_i(p_i)}{H_f(p_f)}. \tag{5}$$

As derived in the Appendix A.5, the network share n_f of firm f in the interior consumption

[7]This selection has the following asymptotic justification. Consider the following best-response dynamics. First, fix an initial value of the vector of network shares $(n_f^0)_{f\in\mathcal{F}}$ such that $n_f^0 > 0$ for all $f \in \mathcal{F}$. Next, for each $t > 0$, update the network share based on the value of network share in the previous iteration $t-1$. Then, the sequence of network shares $\left\{(n_f^t)_{f\in\mathcal{F}}\right\}_{t=0,\dots}$ is obtained. Here, for any $t > 0$, we have

$$\frac{n_f^t}{n_g^t} = \frac{H_f}{H_g}\left(\frac{n_f^{t-1}}{n_g^{t-1}}\right)^\alpha = \left(\frac{H_f}{H_g}\right)^{\sum_{\tau=0}^t \alpha^\tau}\left(\frac{n_f^0}{n_g^0}\right)^{\alpha^t} \to \left(\frac{H_f}{H_g}\right)^{\frac{1}{1-\alpha}} \in (0,\infty) \quad \text{as } t\to\infty. \tag{4}$$

From this observation, we must have the vector of positive network shares as the limit of the best-response dynamics. Thus, the interior consumption equilibrium is the only equilibrium to which the best-response dynamics from any vector of positive network shares.

equilibrium is given by

$$n_f(p) = \frac{\left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}}{H(p)}. \tag{6}$$

Equations (5) and (6) imply that the probability that product $i \in \mathcal{N}_f$ is purchased by a consumer is given by the following equation:

$$s_i(p) = n_f(p)\frac{h_i(p_i)}{H_f(p_f)} = \frac{\left(H_f(p_f)\right)^{\frac{\alpha}{1-\alpha}} h_i(p_i)}{H(p)}. \tag{7}$$

Therefore, the demand for the product $i \in \mathcal{N}_f$ given the profile of prices p has the following form:

$$\begin{aligned} D_i(p) &= \hat{D}_i\left(p_i, H_f(p_f), H(p)\right) \\ &= s_i(p) \times \frac{-h_i'(p_i)}{h_i(p_i)} \\ &= -\frac{\left(H_f(p_f)\right)^{\frac{\alpha}{1-\alpha}} h_i'(p_i)}{H(p)}. \end{aligned} \tag{8}$$

With the CES-class demand and negative price, Roy's identity cannot be used to derive demand. Thus, to allow for the demand with negative prices, we assume that $D_i(p) = \lim_{p_i \to 0} D_i(p) = +\infty$ for all $p_i < 0$.

Finally, the consumer surplus CS is given by the expected indirect utility of consumers, which is given by

$$\begin{aligned} CS(p) &= \log\left(\sum_{f \in \mathcal{F}} \left(H_f(p_f)\right)^{\frac{1}{1-\alpha}} \frac{1}{(H(p))^{\alpha}}\right) \\ &= (1-\alpha)\log H(p). \end{aligned} \tag{9}$$

I put one remark on the form of demand function that is given by equation (8). The demand function for the MNL-class demand system has the form

$$D_i(p) = \frac{\left\{\sum_{j \in \mathcal{N}_f} \exp\left(\frac{a_j - p_j}{\lambda}\right)\right\}^{\delta}}{\sum_{f' \in \mathcal{F}} \left\{\sum_{j \in \mathcal{N}_{f'}} \exp\left(\frac{a_j - p_j}{\lambda}\right)\right\}^{\delta}} \frac{\exp\left(\frac{a_i - p_i}{\lambda}\right)}{\sum_{j \in \mathcal{N}_f} \exp\left(\frac{a_j - p_j}{\lambda}\right)}$$

with $\delta := 1/(1-\alpha)$. This demand function can be regarded as a nested-logit demand function with the set of nests $\{\mathcal{N}_f\}_{f \in \mathcal{F}}$. One difference of this demand function from standard nested-logit demand is that this demand function has $\delta = 1/(1-\alpha) \geq 1$, while standard nested-logit demand functions based on discrete-choice model must have a nest coefficient $\delta \leq 1$. This is because every pair of products should be substitutes in the standard discrete-choice models, and if $\delta > 1$, two products in the same nest is complements when $n_f < (\delta - 1)/\delta$. In this sense, the demand function given by equation (8) allows for complementarity among products and shows that network externalities provide a microfoundation for nested-logit demand functions with nest coefficient δ greater than 1.

3.2. Firm pricing and equilibrium

Each product $i \in \mathcal{N}$ has a constant marginal cost $c_i > 0$ of production. Given the demand system, the profit function of each firm $f \in \mathcal{F}$ is written as a function of the profile of the firm's own prices $p_f = (p_i)_{i \in \mathcal{N}_f}$ and the industry-level aggregator H:

$$\Pi_f(p) = \sum_{i \in \mathcal{N}_f} \hat{D}_i \left(p_i, H_f(p_f), H(p)\right)(p_i - c_i). \tag{10}$$

A pricing game consists of a demand system $(D_i)_{i \in \mathcal{N}}$, a set of firms $\mathcal{F}$, sets of the products of each firm $(\mathcal{N}_f)_{f \in \mathcal{F}}$, and a profile of marginal costs $(c_i)_{i \in \mathcal{N}}$. In the pricing game, firms simultaneously set the prices $p_f := (p_i)_{i \in \mathcal{N}_f}$ of their products, with the payoff function Π_f defined by equation (10). I call a Nash equilibrium of this pricing game as a *pricing equilibrium*. In the following analysis, I often suppress the arguments of functions for the sake of readability.

Arranging the first-order condition for the profit-maximization of each firm f ($\partial \Pi_f / \partial p_i = 0$), the price of product $i \in \mathcal{N}_f$ at firm f's best-response should satisfy the equation

$$\begin{aligned}
&-\frac{h_i''(p_i)}{h_i'(p_i)}(p_i - c_i) \\
&= 1 - \underbrace{\frac{\alpha}{1-\alpha}\frac{\Pi_f}{n_f}}_{\text{network-externality discount}} + \underbrace{\frac{1}{1-\alpha}\Pi_f}_{\text{cannibalization terms}} \\
&=: \mu_f
\end{aligned} \tag{11}$$

for some μ_f. Following Nocke and Schutz (2018b), I call μ_f as the *ι-markup* of firm f. This ι-markup summarizes the pricing incentive of each firm for each product.

As shown in equation (11), ι-markup is decomposed into three factors. The first term, 1, in the second line of the equation (11) is the baseline ι-markup, which would be set under the monopolistic competition. The second term is the downward-pricing pressure to expand the networks. The third term is the upward-pricing pressure of oligopoly due to the internalization of cannibalization effects of the change in the industry-level aggregator. The relative magnitudes of the second and the third terms determine the price level of each firm.

We have $-h_i''(p_i)/h_i'(p_i) = 1/\lambda$ and thus $p_i = c_i + \lambda \mu_f$ in the case of MNL-class demand systems, and $-h_i''(p_i)/h_i'(p_i) = \sigma/p_i$ and thus $p_i = c_i/(1 - \mu_f/\sigma)$ in the case of CES-class demand systems. Using these functional forms, the formula for the firm-level aggregators and profit functions are given by

$$H_f = \begin{cases} T_f \exp(-\mu_f) & \text{in the case of MNL-class demand,} \\ T_f \left(1 - \frac{\mu_f}{\sigma}\right)^{\sigma-1} & \text{in the case of CES-class demand,} \end{cases} \tag{12}$$

and

$$\Pi_f = \begin{cases} n_f \mu_f & \text{in the case of MNL-class demand,} \\ \frac{\sigma-1}{\sigma} n_f \mu_f & \text{in the case of CES-class demand,} \end{cases} \tag{13}$$

where $T_f := \sum_{i \in \mathcal{N}_f} \exp\left(\frac{a_i - c_i}{\lambda}\right)$ for MNL-class and $T_f := \sum_{i \in \mathcal{N}_f} a_i c_i^{1-\sigma}$ for CES-class demand. T_f is called as the "type" of firm f that equals the value of the firm-level aggregator of firm f when it engages in marginal cost pricing. Inserting equations (3), (12), and (13) into the first-order condition (11) shows that the ι-markup μ_f and the network share n_f depends only on its type T_f and the value of the indusry-level aggregator H. Specifically, the first-order condition can be rewritten as

$$1 = \frac{\mu_f}{1-\alpha} \left(1 - \frac{\gamma(T_f)}{H} \exp\left(-\frac{\mu_f}{1-\alpha}\right)\right) \tag{FOC-MNL}$$

and

$$1 = \frac{\mu_f}{\sigma(1-\alpha)} \left(\sigma - \alpha - (\sigma - 1)\frac{\gamma(T_f)}{H} \left(1 - \frac{\mu_f}{\sigma}\right)^{\frac{\sigma-1}{1-\alpha}}\right) \tag{FOC-CES}$$

where $\gamma(x) = x^{\frac{1}{1-\alpha}}$, is the function that amplifies each firm's type through network externalities.

Solving the equation (FOC-MNL) for MNL-class and (FOC-CES) for CES-class demand systems, the ι-markup is obtained as $\mu_f = m(\gamma(T_f)/H, \alpha)$. This implies that all the relevant information for each firm's ι-markup and thus each firm's pricing is summarized in a unidimensional type T_f. This property is called as the "type-aggregation property" (Nocke and Schutz, 2018b), which simplifies the equilibrium and subsequent merger analysis. Using this ι-markup function, we further obtain the network share n_f as

$$n_f = N\left(\frac{\gamma(T_f)}{H}, \alpha\right) := \frac{\gamma(T_f)}{H} \exp\left(-\frac{m\left(\frac{\gamma(T_f)}{H}, \alpha\right)}{1-\alpha}\right), \tag{Share-MNL}$$

and

$$n_f = N\left(\frac{\gamma(T_f)}{H}, \alpha\right) := \frac{\gamma(T_f)}{H} \left(1 - \frac{m\left(\frac{\gamma(T_f)}{H}, \alpha\right)}{\sigma}\right)^{\frac{\sigma-1}{1-\alpha}}, \tag{Share-CES}$$

respectively. It turns out that the function $N(\cdot, \alpha)$ is increasing in the first argument. Thus, a firm f has a large network share n_f either when it has a high type T_f or the value of industry-level aggregator H is small.

Finally, the equilibrium condition for the industry-level aggregator H is that the sum of network shares equals one, that is,

$$\sum_{f \in \mathcal{F}} N\left(\frac{\gamma(T_f)}{H}, \alpha\right) = 1. \tag{14}$$

Solving this equation, the equilibrium value of the industry-level aggregator, H^*, is obtained.

The following lemma summarizes this discussion.

Lemma 1. *For any MNL-class or CES-class demand system, there exists a unique pricing equilibrium where each firm $f \in \mathcal{F}$ sets its price profile $p_f^* = (p_i^*)_{i \in \mathcal{N}_f}$ such that*

$$p_i^* = \begin{cases} c_i + \lambda m\left(\frac{\gamma(T_f)}{H^*}, \alpha\right) & \text{in the case of MNL-class demand,} \\ \frac{c_i}{1 - \frac{m\left(\frac{\gamma(T_f)}{H^*}, \alpha\right)}{\sigma}} & \text{in the case of CES-class demand,} \end{cases}$$

where H^* is the solution to equation (14).

In Appendix A, I provide a more generalized result for the existence and the uniqueness of the pricing equilibrium that incorporates both direct indirect network externalities (Proposition 8). Lemma 1 is one special case of that result.

Before discussing the welfare implication of the mergers in the presence of network externalities, it is worth noting that the definition of the type in this study differs from that in Nocke and Schutz (2018a) in the following sense. In Nocke and Schutz (2018a), type $\mathcal{T}_f^{NS}$ is defined as

$$\mathcal{T}_f^{NS} := \left(\sum_{j \in \mathcal{N}_f} h_j(c_j)\right)^{\delta},$$

where $\delta \in [0, 1]$ is the nest coefficient. Nocke and Schutz (2018a) include this nest coefficient in the definition of types because this emerges from the distribution of the consumers' preference for products, which are unaffected by mergers. By contrast, the definition of the type in this study is $T_f := \sum_{j \in \mathcal{N}_f} h_j(c_j)$ but not $\left(\sum_{j \in \mathcal{N}_f} h_j(c_j)\right)^{1/(1-\alpha)}$ because the nest coefficient $1/(1 - \alpha)$ emerges from network effects, which mergers affect. This difference of the definition of types changes the welfare implication for the mergers without synergies, as shown in the next section.

4. Merger in the Presence of Direct Network Externalities

Based on the equilibrium analysis in the previous section, I proceed to analyze the conditions under which a merger between two firms improves or hurts the consumer surplus. To highlight the impact of network externalities, α is assumed to be strictly positive in the following analysis.

Suppose that two firms f and g with the types T_f and T_g merge to create a new firm M with type T_M. The merger exhibits some technological synergy, which is captured by $\Delta := T_M - T_f - T_g$. The source of technological synergies may be cost reduction, quality improvement, or introduction of new products. It is assumed that such synergies are exogenous primitives of the merger rather than an endogenous choice of the merged entity.

A merger is said to be *CS-increasing* if it increases the equilibrium consumer surplus. Given that the consumer surplus with equilibrium aggregator H^* is given by $(1-\alpha)\log H^*$, the change in consumer surplus resulting from any particular merger can be calculated by the change in the value of equilibrium aggregator H^*. Under this specification of mergers, a merger between two firms is CS-increasing if and only if

$$N\left(\frac{\gamma(T_f+T_g+\Delta)}{H^*},\alpha\right) \geq N\left(\frac{\gamma(T_f)}{H^*},\alpha\right)+N\left(\frac{\gamma(T_g)}{H^*},\alpha\right) \tag{15}$$

holds, where H^* is the pre-merger equilibrium aggregator.[8] Thus, the merger is CS-increasing if and only if the post-merger network share of the merged entity exceeds the pre-merger total network share of merging parties.

From the condition (15) and the fact that the network share function is increasing in the type, the merger between two firms is CS-increasing if and only if $\Delta \geq \hat{\Delta}$, where $\hat{\Delta}$ satisfies

$$N\left(\frac{\gamma(T_f+T_g+\hat{\Delta})}{H^*},\alpha\right) = N\left(\frac{\gamma(T_f)}{H^*},\alpha\right)+N\left(\frac{\gamma(T_g)}{H^*},\alpha\right). \tag{16}$$

This value $\hat{\Delta}$ is the *CS-neutral technological synergy* required for a merger between two firms. Let $\hat{T}_M := T_f + T_g + \hat{\Delta}$ be the type of the merged entity with CS-neutral technological synergy. The larger the CS-neutral technological synergies are, the more likely it is that mergers with a given value of technological synergy reduce consumer surplus. In this sense, the value of CS-neutral technological synergy can be interpreted as a criterion for approving mergers.

The subsequent sections examine the welfare properties of mergers in the presence of direct network externalities using the notion of CS-neutral technological synergies

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4.1. Merger involving a small firm

First, I show that in the presence of firm-level network effects, acquisition of a sufficiently small firm is CS-increasing without technological synergy, that is, $\hat{\Delta} < 0$.

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[8]This condition is derived as follows. Suppose that equation (15) holds with strict inequality, then, we have

$$\sum_{f'\in\mathcal{F}} N\left(\frac{\gamma(T_{f'})}{H^*},\alpha\right)+N\left(\frac{\gamma(T_f+T_g+\Delta)}{H^*},\alpha\right) > 1.$$

Since the function $N(x,\alpha)$ is decreasing in x, the value of post-merger equilibrium aggregator is greater than H^*. Conversely, if equation (15) does not hold, we have

$$\sum_{f'\in\mathcal{F}} N\left(\frac{\gamma(T_{f'})}{H^*},\alpha\right)+N\left(\frac{\gamma(T_f+T_g+\Delta)}{H^*},\alpha\right) < 1.$$

Thus, the value of post-merger equilibrium aggregator is smaller than H^*. Thus, a merger improves consumer surplus if and only if equation (15) holds.

Proposition 1. *(Merger involving a small firm) Consider a merger between firm f and firm g. If one of the merging parties is sufficiently small, the merger is CS-increasing without technological synergy. That is, there exists $\bar{T}_f$ such that for any $T_f < \bar{T}_f$, $\hat{\Delta} < 0$.*

Proof. In Appendix B.1. □

In the presence of network externalities, a small firm is disadvantaged in selling the products because of a lack of installed base. When this firm is acquired by a large firm, the acquirer can leverage its own network to sell the small firm's products, which also benefits consumers who wish to access the small firm's products. Further, the contribution of the acquisition of this small firm to the increase in the market power of the merged entity is negligible. Thus, overall, the former benefit from an increase in the access to the products of the small firm dominates, and the consumer surplus increases. This result is in sharp contrast with the result of Nocke and Schutz (2018a) that any merger without synergy reduces the consumer surplus.

4.2. Merger from symmetric oligopoly

Next, I examine the welfare effects of mergers in an originally symmetric oligopoly. The symmetric environment is well suited to illustrate the trade-off between the benefit from network expansion and the cost of increased market power in a simplest manner.

Consider a symmetric oligopoly with $|\mathcal{F}|$ firms with the same types T. In the symmetric oligopoly, the equilibrium market share of each firm is given by $1/|\mathcal{F}|$. Thus, the value of the equilibrium aggregator is derived by solving

$$N\left(\frac{\gamma(T)}{H^*}, \alpha\right) = \frac{1}{|\mathcal{F}|}$$

Based on the value of the pre-merger equilibrium aggreagtor H^*, consider a merger between two firms that does not generate any synergy (i.e., $\Delta = 0$). The merger improves the consumer surplus if and only if

$$N\left(\frac{\gamma(2T)}{H^*}, \alpha\right) \geq \frac{2}{|\mathcal{F}|} \tag{17}$$

The right-hand side of this inequality does not depend on the value of α. Further, the market share of the merged entity increases with α because the merged entity has the largest network. These jointly imply that the merger is CS-increasing if α is above a certain critical value. The next proposition formalizes this discussion.

Proposition 2. *(Merger from symmetric oligopoly) Suppose that all firms are symmetric so that $T_f = T$ for all $f \in \mathcal{F}$. Then;*

1. *there exists a critical value of the magnitude of direct network externalities above which a merger between two firms are CS-increasing without technological synergy. That is, there exists $\hat{\alpha}$ such that $\hat{\Delta} < 0$ if and only if $\alpha > \hat{\alpha}$;*

2. *$\hat{\alpha}$ decreases with the number of firms $|\mathcal{F}|$.*

Proof. In Appendix B.2. □

As discussed earlier, when the magnitude of network effects is greater than some critical value, the merger between two firms improves the consumer surplus. Further, as the number of firms increases, the critical value of the magnitude of network effects reduces, because the increase in the market power due to the merger becomes less important as the market is more competitive before the merger, which decreases the magnitude of the network effects required to offset the consumer harm due to market power.

Overall, as long as the firms are symmetric, the presence of network externalities provides a justification of mergers. In this sense, to some extent, the network externalities can be regarded as a form of synergies accompanying mergers.

4.3. Technological synergies and network effects

In the preceding analyses, I have discussed the cases where mergers without technological synergies can be CS-increasing. These results indicate that network externalities can be regarded as a form of synergies accompanying mergers. This section analyzes the relation between the CS-neutral technological synergies and the magnitude of network externalities and examines the extent to which the network externalities can be regarded as a form of synergies.

For the tractability of analysis, I confine the analysis to the case with MNL-class demand systems. The MNL-class demand systems have an advantage that the market share function $N((\gamma(T_f)/H, \alpha)$ depends on α only through the value of $\gamma(T_f)/H$. Specifically, the network share function can be written as $N((\gamma(T_f)/H, \alpha) = N_0((\gamma(T_f)/H))$ where $N_0(x) := N(x, 0)$.[9] From this observation, equation (16) can be simplified as

$$N_0\left(\frac{\gamma(T_f + T_g + \hat{\Delta})}{H^*}\right) = N_0\left(\frac{\gamma(T_f)}{H^*}\right) + N_0\left(\frac{\gamma(T_g)}{H^*}\right). \tag{18}$$

Using this condition, I analyze how CS-neutral technological synergies vary with respect to firm sizes and the magnitude of direct network externalities.

First, I examine how the size of merging parties affects the technological synergies required to justify mergers. As shown in Proposition 1, when one of the merging parties is small, then the merger makes it easier to sell the product to consumers through the expansion of networks, which improves the consumer surplus. However, as the merging parties become large, such benefit from network expansions are offset by the accompanying increase in the market power, which hurts consumers. These jointly lead to the conjecture that the CS-neutral synergies are negative as long as the merging parties are small, and then become positive when the sizes

[9] To see this, note that from equation FOC-MNL, $\mu_f/(1-\alpha)$ is determined only through $\gamma(T_f)/H$. Letting this value denoted as $\mu_f/(1-\alpha) =: \tilde{m}_0(\gamma(T_f)/H)$, we have the network share function by

$$N\left(\frac{\gamma(T_f)}{H}, \alpha\right) = \frac{\gamma(T_f)}{H} \exp\left\{-\tilde{m}_0\left(\frac{\gamma(T_f)}{H}\right)\right\} = N\left(\frac{\gamma(T_f)}{H}, 0\right).$$

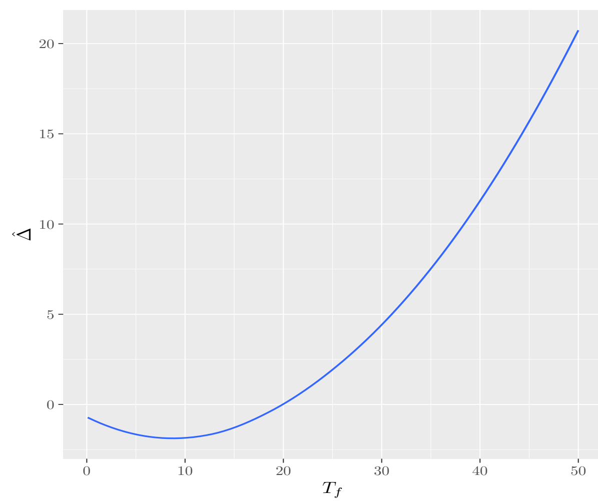


Figure 1: The relation between the size of merging parties and the size of technological synergies required for a merger to be CS-increasing.

of the merging parties exceeds certain a threshold. The following proposition formalizes such intuition, and Figure 1 illustrates this result.

Proposition 3. *(Firm sizes and technological synergies) For any merger between firm f and g with pre-merger network shares N_f and N_g and pre-merger equilibrium aggregator H^*, there exists a critical value of pre-merger joint network share $\tilde{N}$ such that*

1. *If $N_f + N_g \le \tilde{N}$, $\hat{\Delta}$ decreases with T_f;*
2. *If $N_f \ge \tilde{N}$, $\hat{\Delta}$ increases with T_f;*
3. *If $N_f < \tilde{N} < N_f + N_g$, then $\hat{\Delta}$ increases with T_f if and only if N_g is greater than some threshold $\tilde{N}_g(N_f)$, which decreases with N_f.*

Proof. In Appendix B.3 □

Proposition 3 and the fact that $\hat{\Delta} = 0$ for $T_f = 0$ jointly lead to the following corollary.

Corollary 1. *Consider a merger between firm f and g with pre-merger network shares N_f and N_g. Given the value of the pre-merger equilibrium aggregator, there exists $\bar{N}(N_g)$ such that $\hat{\Delta} < 0$ if and only if $N_f < \bar{N}_f(N_g)$.*

This corollary implies that as long as the pre-merger network shares of the merging parties are below certain critical values, the merger between them is CS-increasing without technological synergies, which generalizes Proposition 1.

Next, I examine how the magnitude of network externalities affects the technological synergies required to improve the consumer surplus. To this end, I introduce some terminologies to describe the firms' sizes, based on the relation between the magnitude of network externalities and the pre-merger network shares. Using the Implicit Function Theorem, we have the following relation:

$$\frac{d}{d\alpha}\left(\frac{\gamma(T_f)}{H^*}\right) = \frac{1}{(1-\alpha)^2}\frac{\gamma(T_f)}{H^*}\frac{\sum_{f'\in\mathcal{F}}(\log T_f - \log T_{f'})N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)}{\sum_{f'\in\mathcal{F}} N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)} \tag{19}$$

From this equation, the following lemma is obtained.

Lemma 2. *(Network effects and market share) For any type profile $\{T_f\}_{f\in\mathcal{F}}$, there is a threshold value T^* such that*

$$\frac{d}{d\alpha}\left(\frac{\gamma(T_f)}{H^*}\right) \geq 0 \tag{20}$$

if and only if $T_f \geq T^*$. Consequently, $N_0(\gamma(T_f)/H^*)$ increases with α if and only if $T_f \geq T^*$.

This result is an example of familiar positive-feedback effects of network externalities. Network effects expand the market shares of firms with greater market shares and shrink that of firms with smaller market share. The threshold type T^* stands for the critical value defining the direction in which the positive feedback effects influence the market shares. I call the firms with $T_f > T^*$ as *strong firms* and the firms with $T_f < T^*$ as *weak firms*.

Based on the definitions of strong firms and weak firms, I analyze the impact of network effects on CS-neutral technological synergies. Using the Implicit Function Theorem, the change in $\hat{\Delta}$ according to the change in α can be written as

$$\frac{d\hat{\Delta}}{d\alpha} = \left\{\frac{d}{d\Delta}N_0\left(\frac{\gamma(\hat{T}_M)}{H^*}\right)\right\}^{-1}\left\{\frac{d}{d\alpha}N_0\left(\frac{\gamma(T_f)}{H^*}\right) + \frac{d}{d\alpha}N_0\left(\frac{\gamma(T_g)}{H^*}\right) - \frac{d}{d\alpha}N_0\left(\frac{\gamma(\hat{T}_M)}{H^*}\right)\right\} \tag{21}$$

The following proposition characterizes how the impacts of network externalities on CS-neutral technological synergies vary with the sizes of merging parties.

Proposition 4. *(Network externalities and technological synergies) Consider a merger between firms f and g with types T_f and T_g that lead to pre-merger equilibrium network shares N_f and N_g.*

1. *If both f and g are weak, then $\hat{\Delta}$ decreases with α.*
2. *If f is strong and g is weak, then there exists $\hat{N} \in (0,1)$ such that if $N_f + N_g < \hat{N}$, then $\hat{\Delta}$ decreases with α.*
3. *If both f and g are strong and $N_f + N_g$ is close to 1. Then $\hat{\Delta}$ increases with α.*

Proof. In Appendix B.4. □

The underlying intuition of the first part of Proposition 4 is similar to that of Proposition 1. Weak firms fail to leverage the network effects due to their small market shares. The greater the magnitude of the network effects, the more serious is the failure of the weak firms to internalize the network externalities. In such cases, the merger softens this failure to internalize the network externalities. Therefore, the merger between weak firms is more desirable as the magnitude of network effects increases. A similar argument partially extends to the case where one firm in the merging party is strong and the other is weak, which leads to the second part of Proposition 4.

The impact of network effects on mergers between strong firms is ambiguous. When both of the merging parties are strong, the demand-side scale economy may not be sufficient to compensate an increase in the markups caused by a greater concentration. When the joint market share of the two firms is too large, the merged entity has an extremely strong market power. An increase in the consumer gain from a further increase in the magnitude of network effects is offset by an increase in the markup of the merged entity. Therefore, when the joint market share is sufficiently large, a greater magnitude network effects requires greater synergies for mergers between large firms to improve consumer welfare. This result is consistent with the concern raised by EU competition authority regarding the potential harm to the competition in the merger between Facebook and WhatsApp.[10] Figure 2 shows numerical examples of Proposition 4 in an industry with 12 firms, one of which, f' has $T_{f'} = 25$, another has $T_{f'} = 20$, and the remaining 10 firms have $T_{f'} = 5$.

In summary, the presence of network externalities makes mergers more likely to improve consumer surplus when the merging parties are small, or the firms in the industry are symmetric. However, as the size of the merging parties become larger relative to the size of the industry, the presence of network externalities makes mergers more likely to harm consumers.

5. Acquisition of Innovative Entrants

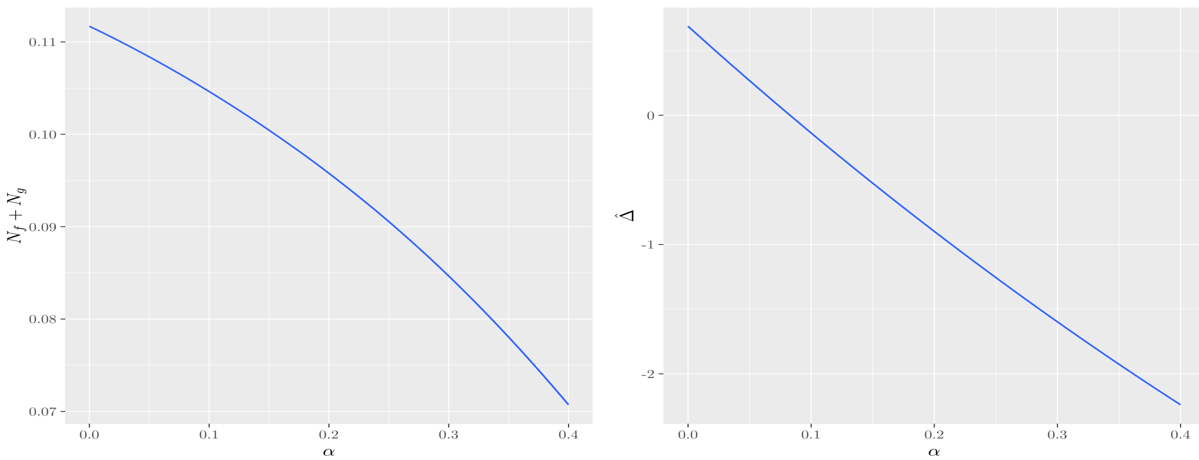
This section considers an acquisition of an innovative entrant by an incumbent and compares the innovation incentives of the incumbents and the entrant, which is studied by Cunningham et al. (2018). In the theoretical part of their analysis, Cunningham et al. (2018) show that the incumbent always has a smaller incentive to continue the innovative projects than the entrant does because of the "replacement effect" but might nevertheless have an incentive to acquire the innovative entrant just to eliminate the future competitor. Such an acquisition is called *killer acquisition*.

By contrast, the presence of network externalities modifies the above argument. Specifically, when the scale of the innovative entrant is not large, the incumbent has a greater incentive to incur costs to continue the project because of the demand-side scale economies. Then, there is a case where the incumbent would continue the project, but the entrant would not, in which case the only way to continue the project is to approve the acquisition of the entrant.

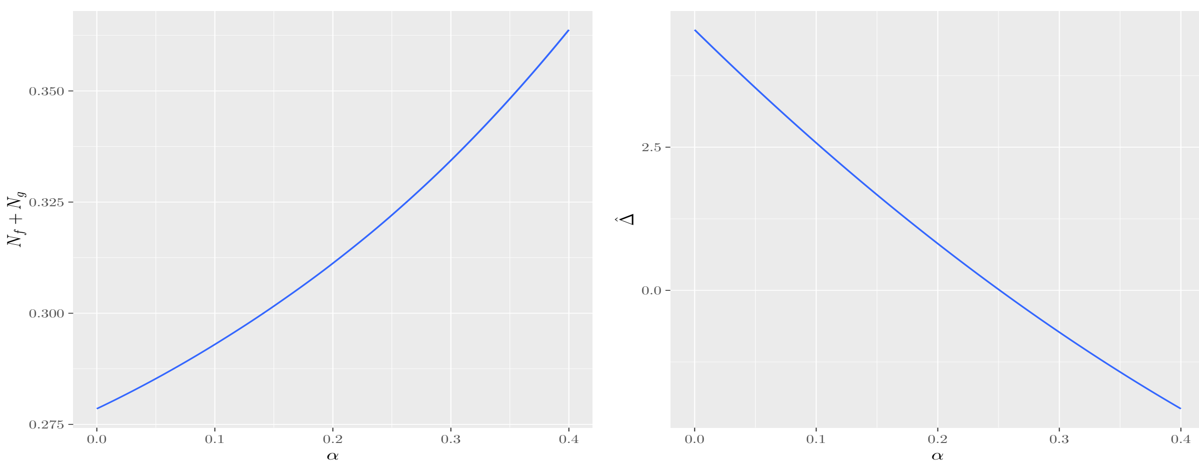
This leads to a policy implication regarding the merger control and innovation. Too stringent merger control may reduce the entrepreneurs' incentive to start new projects, because a merger may be the sole method to make a profit. Indeed, for many startups, one of their final objectives is to be acquired by large tech companies in return for a large payment.

Consider the following game. There is a set $\mathcal{F}$ of incumbents. The demand system is given by MNL-class demand. Each incumbent $f \in \mathcal{F}$ has its type T_f. There is also an innovative potential

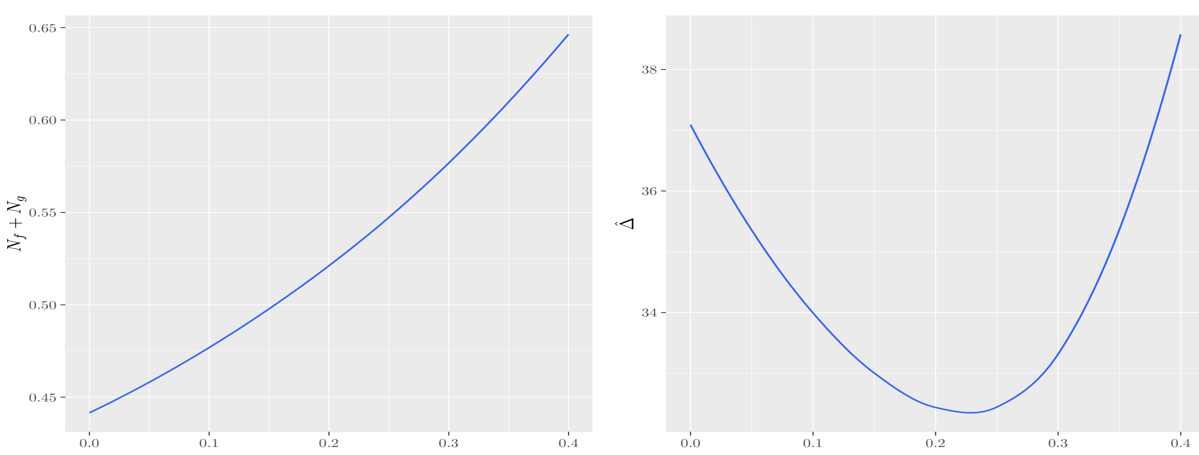
[10] Case No COMP/M.7217 - FACEBOOK/ WHATSAPP



(a) Weak firms ($T_f = T_g = 5$)



(b) Strong and weak firms ($T_f = 25$, $T_g = 5$)



(c) Strong firms ($T_f = 25$, $T_g = 20$)

Figure 2: The effects of direct network externalities α on the joint network share $N_f + N_g$ and the required technological synergies $\hat{\Delta}$ for merging parties f and g. In this example, there are one firm f' with $T_{f'} = 25$, one firm with $T_{f'} = 20$, and 10 firms with $T_{f'} = 5$.

entrant E who has a project that succeeds with probability $\rho \in (0, 1)$. If the project succeeds, it generates a new product with type T_E. Among the incumbents, there is one incumbent $I \in \mathcal{F}$ interested in acquiring the project of the potential entrant E.

Given this environment, consider the following three-stage game.

1. Whenever feasible, firm I offers a take-it-or-leave-it offer to firm E that specifies the payment to acquire the project. If offered, firm E chooses whether to accept it.

2. The firm that owns the project decides whether to continue or liquidate the project.
   - If the project is continued, it takes cost K and succeeds with probability ρ.
   - If the project is liquidated, it generates the liquidation value L.

3. Depending on the project's success and the owner, three cases emerge.
   - If the project fails, the set $\mathcal{F}$ of firms play the corresponding pricing game. Each firm $f \in \mathcal{F}$ has type T_f.
   - If the project succeeds and firm I owns the project, the set $\mathcal{F}$ of firms play the corresponding pricing game. Each firm $f \in \mathcal{F} \setminus \{I\}$ has type T_f, and firm I has type $T_I + T_E$.
   - If the project succeeds and firm E owns the project, the set $\mathcal{F} \cup \{E\}$ of firms play the corresponding pricing game. Each firm $f \in \mathcal{F} \cup \{E\}$ has type T_f.

Let H_0, H_I, and H_E be the equilibrium values of the aggregators in the cases where (1) project has failed, (2) project has succeeded and I owns it, and (3) project has succeeded and E owns it.

The expected profit of the incumbent from continuing the project is given by

$$\rho N_0 \left(\frac{\gamma(T_I + T_E)}{H_I} \right) m \left(\frac{\gamma(T_I + T_E)}{H_I}, 0 \right) + (1 - \rho) N_0 \left(\frac{\gamma(T_I)}{H_0} \right) m \left(\frac{\gamma(T_I)}{H_0}, 0 \right) - K,$$

whereas the expected profit of the incumbent from liquidating the project is given by

$$N_0 \left(\frac{\gamma(T_I)}{H_0} \right) m \left(\frac{\gamma(T_I)}{H_0}, 0 \right) + L.$$

Thus, the incumbent continues the project if and only if

$$\Delta\pi^I := \rho \left\{ N_0 \left(\frac{\gamma(T_I + T_E)}{H_I} \right) m \left(\frac{\gamma(T_I + T_E)}{H_I}, 0 \right) - N_0 \left(\frac{\gamma(T_I)}{H_0} \right) m \left(\frac{\gamma(T_I)}{H_0}, 0 \right) \right\} \geq L + K \tag{22}$$

The expected profit of the entrant from continuing the project is given by

$$\rho N_0 \left(\frac{\gamma(T_E)}{H_E} \right) m \left(\frac{\gamma(T_E)}{H_E}, 0 \right) - K,$$

whereas the expected profit of the entrant from liquidating the project is given by L. Thus, the incumbent continues the project if and only if

$$\Delta\pi^E := \rho N_0 \left(\frac{\gamma(T_E)}{H_E}\right) m \left(\frac{\gamma(T_E)}{H_E}, 0\right) \geq L + K \tag{23}$$

The following proposition highlights that, when T_E is small, $\Delta\pi^I > \Delta\pi^E$ can hold, in which case for some values of K, the project is continued only if it is owned by the incumbent.

Proposition 5. *(Innovation incentives) The following statements hold:*

1. *If T_E is sufficiently small, then the incumbent has a greater incentive to continue the project, that is, $\Delta\pi^I > \Delta\pi^E$ holds.*

2. *If T_E is sufficiently large, then the incumbent has a smaller incentive to continue the project, that is, $\Delta\pi^I < \Delta\pi^E$ holds.*

Proof. In Appendix B.5. □

If the size of innovation is small relative to the industry, the reverse of killer acquisition may hold. An entrant with small innovation cannot break-even by just entering into the market, but the incumbent can gain by purchasing such an innovation. This is because the entrant cannot attract consumers to the new product because of the lack of the network, while the incumbent can leverage its installed base to attract consumers. Formally, there exist a pair of innovation cost K and liquidation value L such that the incumbent continues the innovation, but the entrant does not.

However, if both the entrant and incumbent are large relative to the industry, the killer acquisition may again emerge by the same logic as Cunningham et al. (2018), which is shown in the second part of the proposition.

For completeness, I close the model when T_E is sufficiently small. When T_E is sufficiently small, $\Delta\pi^I > \Delta\pi^E$ holds by Proposition 5. In such cases, firm I offers a payment $\rho\Delta\pi^E - K - L$ and acquires the project whenever feasible. However, if the merger is expected to be blocked and $K + L \in (\rho\Delta\Pi^E, \rho\Delta\pi^E)$ holds, firm E will not continue the project, and thus the innovation is not completed.

6. Merger in Two-Sided Markets

Finally, I analyze the mergers in the presence of indirect network externalities, with particular interest in two-sided markets. The focus is how structures of firm's sizes on two sides of markets affect the welfare properties of mergers. I first examine how two-sidedness affects the price structures and welfare properties of equilibria. Then, based on the equilibrium analysis, I show how two-sidedness affects the welfare properties of mergers. A detailed derivation of equilibrium is relegated to the general analysis in Appendix A.

6.1. Equilibrium analysis

Setup Consider an industry with two sides of markets A and B with a set $\mathcal{N}^J$ of imperfectly substitutable products on side $J \in \{A, B\}$ produced by the set of firms $\mathcal{F}$. Each firm $f \in \mathcal{F}$ produces the set $\mathcal{N}_f^J$ of products on side J, where $\mathcal{N}_f^J \cap \mathcal{N}_{f'}^J = \emptyset$ for $f \neq f'$ and $\bigcup_{f\in\mathcal{F}} \mathcal{N}_f^J = \mathcal{N}^J$.

There is a mass of consumers on side $J \in \{A, B\}$ who derive firm-level indirect network externalities from the purchase of each product $i \in \mathcal{N}_f^J$, which depends on the number of consumers on the other side who purchase the products of firm f. Specifically, each consumer $z \in [0, 1]$ on side $J \in \{A, B\}$ yields the indirect subutility from the purchase of product $i \in \mathcal{N}_f^J$ by

$$\log h_i^J(p_i) + \beta_J \log n_f^I + \varepsilon_{iz}^J \tag{24}$$

where $\log h_i^J(p_i)$ is the stand-alone indirect subutility, β_J is the magnitude of the indirect network externalities, n_f^I is the number of consumers on side $I \neq J$ who purchase the product of firm f, and ε_{iz}^J is an idiosyncratic taste shock that follows an i.i.d. type-I extreme-value distribution. In this section, I assume that demand system is MNL-class, $h_i^J(p_i) = \exp((a_i - p_i)/\lambda^J)$, and that only consumers on side A enjoy indirect network externalities from consumers on side B, that is, $\beta_A = \beta \in (0, 1)$, and $\beta_B = 0$. The situation that suits this assumption would be media platforms where firms are newspapers or online content services, consumers on side A are advertisers who benefit from the viewers who are consumers on side B. In some industries, advertisements are considered a nuisance but sometimes helpful to consumers. Another example is an industry where firms are advertising networks, consumers on side A are advertisers, and consumers on side B are publishers who allow advertising networks to display ads. The insights from the results in this setting would apply to the setting where consumers on both sides of the market derive indirect network externalities as long as there is asymmetry in network externalities between two sides of the markets.

Consumers on each side have no outside option and single-home so that they purchase one product that gives the highest level of utilities. Defining firm-level subaggregator on side J as $H_f^J(p_f) = \sum_{i\in\mathcal{N}^J} h_i^J(p_i)$, consumers' optimal choice of products leads to the network shares

$$n_f^A = \frac{H_f^A\left(p_f^A\right)\left(H_f^B\left(p_f^B\right)\right)^{\beta}}{H^A(p)}, \tag{25}$$

and

$$n_f^B = \frac{H_f^B\left(p_f^B\right)}{H^B(p)}, \tag{26}$$

where

$$H^A(p) = \sum_{f\in\mathcal{F}} H_f^A\left(p_f^A\right)\left(H_f^B\left(p_f^B\right)\right)^{\beta}$$

is the industry-level aggregator on side A, and

$$H^B(p) = \sum_{f \in \mathcal{F}} H_f^B\left(p_f^B\right)$$

is the industry-level aggregator on side B. Finally, the demand for product $i \in \mathcal{N}_f^J$ is given by

$$\hat{D}^J\left(p_i, H_f^J\left(p_f^J\right), H_f^I\left(p_f^I\right), H^J(p)\right) = n_f^J \times \frac{-(h_i^J)'(p_i)}{H_f^J(p_f^J)}. \tag{27}$$

Given the above demand function, each product $i \in \mathcal{N}^J$ has a constant marginal cost $c_i > 0$ of production, and firm f's profit $\Pi_f(p)$ is given by the sum of the profit on two sides of the markets $\Pi_f(p) = \Pi_f^A(p) + \Pi_f^B(p)$, where the profit on side J is given by

$$\Pi_f^J(p) = \sum_{i \in \mathcal{N}_f^J} \hat{D}^J\left(p_i, H_f^J\left(p_f^J\right), H_f^I\left(p_f^I\right), H^J(p)\right)(p_i - c_i). \tag{28}$$

In the pricing game, each firm f chooses its price profile $p_f = (p_i)_{i \in \mathcal{N}_f^A \cup \mathcal{N}_f^B}$ to maximize $\Pi_f(p)$. The Nash equilibrium of the pricing game is called pricing equilibrium.

Equilibrium and welfare analysis Given the significance of the properties of equilibrium pricing and welfare, I examine these properties first.

As in the case with direct network externalities, there is the common ι-markup property; at the optimum, each firm's pricing of each product on each side $J \in \{A, B\}$ is summarized by an ι-markup μ_f^J which is given by

$$-\frac{\left(h_i^J\right)''(p_i)}{\left(h_i^J\right)'(p_i)}(p_i - c_i) = \mu_f^J \text{ for all } i \in \mathcal{N}_f^J,$$

for each $J \in \{A, B\}$. In the case of MNL-class demand system, it turns out that ι-markups μ_f^A and μ_f^B are given by the solution to the following system of equations

$$1 - \left(1 - n_f^A\right)\mu_f^A = 0, \tag{29}$$

and

$$1 - \left(1 - n_f^B\right)\mu_f^B - \beta\frac{n_f^A}{n_f^B} = 0, \tag{30}$$

where network shares n_f^A and n_f^B are given by

$$n_f^A = \frac{T_f^A\left(T_f^B\right)^\beta}{H^A}\exp\left(-\mu_f^A - \beta\mu_f^B\right), \quad n_f^B = \frac{T_f^B}{H^B}\exp\left(-\mu_f^B\right), \tag{31}$$

and $T_f^J := \sum_{f \in N_f^J} h_i^J(c_i)$ is the type of firm $f \in \mathcal{F}$ on side $J \in \{A, B\}$. Let m^A and m^B be the solution to this system of equations, as functions of (T_f^A, T_f^B, H^A, H^B). Then, plugging $\mu_f^J = m^J$ into equation (31), the network share functions $N^A(T_f^A, T_f^B, H^A, H^B)$ and $N^B(T_f^A, T_f^B, H^A, H^B)$ are obtained. Finally, the equilibrium condition for the industry-level aggregators H^A, H^B is given by

$$\sum_{f \in \mathcal{F}} N^J\left(T_f^A, T_f^B, H^A, H^B\right) = 1, \text{ for } J \in \{A, B\}. \tag{32}$$

Appendix A shows the uniqueness of pricing equilibrium in this environment. Let H^{A*} and H^{B*} be the equilibrium value of the aggregators.

Before the merger analysis, it is useful to understand the nature of price structure in the two-sided markets in this framework. One particular interest is how an increase in the type on one side affects the pricing incentive on the other side, and the resulting market share.

First, I introduce some terminology to describe the price structure of each firm. Equations (29) and (30) show that given the fixed network shares, each firm has an incentive to lower prices on side B than on side A because the firms can attract more consumers and charge higher prices on side A by providing indirect network externalities by attracting consumers on side B. Thus, consumers on side B are “subsidized" through lower prices, while consumers on side A are “subsidizing" through paying relatively higher prices. Based on this observation, I call side A as *subsidizing segment* and side B as *subsidized segment*, which follows the terminology used by Rochet and Tirole (2003).

The following lemma illustrates the relationship between the type of a firm on each side and its ι-markups and network shares on two sides of markets.

Lemma 3. *The following statements hold:*

1. *Each firm's price level on subsidizing segment increases with the firm's type on both segments. That is, m^A is increasing in T_f^A and T_f^B.*

2. *Each firm's price level on subsidized segment increases with the firm's type on that segment, but decreases with the firm's type on the subsidizing segment. That is, m^B is decreasing in T_f^A and increasing in T_f^B.*

3. *Each firm's network share on each segment increases with the firm's type on both segments. That is, both N^A and N^B are increasing in T_f^A and T_f^B.*

Proof. In Appendix B.6. □

This lemma characterizes how the price structure and the network shares of each firm on each side are related to its types on two sides of the markets. The first part of the lemma shows how the ι-markup on subsidizing segment, side A, varies with its type on each side. Given that the consumers on side A benefit from the indirect network externalities from side B, increases in T_f^A and T_f^B both expand the network share of firm f on side A through an increase in either the productivity or the benefit of indirect network externalities. Such an increase in network size leads the firm to set a higher markup on side A. The second part of the lemma shows how the ι-markup on subsidized segment, side B, varies with T_f^A and T_f^B. An increase in T_f^B simply increases the market share on side B and thus increases the markup on side B. By contrast, an increase in T_f^A enlarges the firm's customer base and increases the markup on side A, which increases the firm's incentive to further expand the customer base on side A. Therefore, the firm lowers the ι-markups on side B to further subsidize consumers on side B. Finally, the changes in the network shares depend on the direct effect of a change in types and the indirect effect of a change in ι-markup. It turns out that the former dominates the latter whenever they conflict, and thus the network shares on both sides increase with firm's type on each side.

Next, as a preliminary for examining the welfare effects of mergers, consider the relationship between equilibrium consumer surplus and firms' types on each side. The equilibrium consumer surplus on each side is given by

$$CS^{A*} = \log H^{A*} - \beta \log H^{B*} \text{ and } CS^{B*} = \log H^{B*}. \tag{33}$$

The aggregate consumer surplus is given by

$$CS^{*} = CS^{A*} + CS^{B*} = \log H^{A*} + (1-\beta) \log H^{B*}. \tag{34}$$

These equations show that the equilibrium consumer surplus on each side can be calculated using the values of equilibrium aggregators H^{A*} and H^{B*}. Thus, by characterizing the relation between the equilibrium aggregators and the primitives such as firms' types on the two sides of markets, the determinants of equilibrium consumer surplus can be analyzed. The following result shows the relation between equilibrium aggregators and firms' types.

Proposition 6. *(Firm types and equilibrium aggregators) Consider a pricing equilibrium with equilibrium aggregators H^{A*}, H^{B*}, and equilibrium network shares $(n_f^J)_{f\in\mathcal{F}}$ for $J = A, B$. The following statements hold.*

1. *Both H^{A*} and H^{B*} increase with T_f^B.*
2. *H^{A*} increases with T_f^A.*
3. *There exists $\bar{n}_f^A$ such that H^{B*} increases with T_f^A if $n_f^A \geq \bar{n}_f^A$. Otherwise, H^{B*} may decrease with T_f^A.*

Proof. In Appendix B.7 □

This proposition states that both H^{A*} and H^{B*} increase with the type T_f^B of firm f on subsidized segment (side B), which implies that the aggregate consumer surplus that is defined by the sum of consumer surplus on the two sides increases with T_f^B. However, an increase in type T_f^A of firm f on the subsidizing segment (side A) might decreases H^{B*}, while it always increases H^{A*}. This occurs in the following scenario. Suppose that the type T_f^A of firm f on side A increases. This increases the share of firm f on side A, which leads to a lower price on side B to attract more consumers on side A. However, it decreases the share of other firms on side A, which leads to higher prices on side B due to the reduced incentives of firms to subsidize side B. The consumer surplus on side B may decrease if the former gain is not large enough to offset the latter loss. When firm f is small, the latter loss can dominate and eventually hurt consumers on side B in aggregate.

This proposition leads to the following implications for merger controls. Unlike the one-sided markets, synergies may not always benefit consumers on every side, especially when such synergies are generated on the subsidizing segments, such as advertisers' side in media platforms. By contrast, synergies generated on subsidized segments, such as eyeballs, are likely to improve consumer surplus in aggregate. In this sense, it may be more conservative to focus on the synergies generated on subsidized segments when faced with a merger between two-sided platforms.

6.2. CS-neutral synergies for mergers in two-sided markets

Based on the equilibrium analysis, I analyze mergers in two-sided markets. Characterizing the consumer-surplus-oriented merger policy with respect to the synergies is challenging, because the consumer surplus can decrease with the type of firm, as shown in Proposition 6. Thus, it is often difficult to characterize the consumer-surplus effects of mergers in two-sided markets using simple threshold types. Nonetheless, Proposition 6 implies that some threshold property in the synergies generated on subsidized segment can be obtained as follows.

Consider a merger between firm f and firm g. Let M be the merged entity with the type $(T_M^A, T_M^B) = (T_f^A + T_g^A + \Delta^A, T_f^B + T_g^B + \Delta^B)$. I call Δ^A and Δ^B as the technological synergy on side A and side B, respectively. A merger is *CS-increasing* if the equilibrium value of CS is greater than that before the merger. The following result shows, as a corollary of Proposition 6, that for any merger with given synergy on side B, the merger is CS-increasing if and only if the synergy on side B is above certain threshold.

Corollary 2. *Consider a merger between firms f and g. For any technological synergy Δ^A on side A, there exists $\bar{\Delta}^B(\Delta^A)$ such that the merger is CS-increasing if and only if $\Delta^B > \bar{\Delta}^B(\Delta^A)$.*

Unfortunately, characterizing the condition under which mergers improve the consumer surplus, that is, the shape of $\bar{\Delta}^B(\Delta^A)$, is complicated because the synergies on side A might reduce the consumer surplus on side B. By contrast, it turns out that characterizing the mergers that leave consumer surplus *on both sides* unchanged is relatively easy, which gives a clean insight on the welfare properties of mergers in two-sided markets. I say that a merger is *strictly CS-neutral* if the equilibrium value of consumer surplus on both sides, CS^{A*} and CS^{B*}, are unchanged as a result of the merger. A merger is strictly CS-neutral if and only if two equations holds;

$$\begin{aligned} N^A\left(T_M^A, T_M^B, H^{A*}, H^{B*}\right) &= N^A\left(T_f^A, T_f^B, H^{A*}, H^{B*}\right) + N^A\left(T_g^A, T_g^B, H^{A*}, H^{B*}\right), \\ N^B\left(T_M^A, T_M^B, H^{A*}, H^{B*}\right) &= N^B\left(T_f^A, T_f^B, H^{A*}, H^{B*}\right) + N^B\left(T_g^A, T_g^B, H^{A*}, H^{B*}\right), \end{aligned} \tag{35}$$

where H^{A*} and H^{B*} are the pre-merger aggregators.

The next lemma shows that there is a unique pair of technological synergies that is strictly CS-neutral.

Lemma 4. *For any merger between firms f and g, there exists a unique pair of technological synergies $(\hat{\Delta}^A, \hat{\Delta}^B)$ such that the merger is strictly CS-neutral and thus satisfies the condition (35).*

Proof. In Appendix B.8. □

The pair of technological synergies $(\hat{\Delta}^A, \hat{\Delta}^B)$ defined in equation (35) has a clear and tractable characterization, which is similar to the condition in one-sided markets shown in (16). Further, this pair of synergies has the following normative interpretation. Whenever $(\Delta^A, \Delta^B) < (\hat{\Delta}^A, \hat{\Delta}^B)$, consumers surplus on at least one side of the markets is reduced as a result of merger. Thus, the technological synergies for a strictly CS-neutral merger can be treated as a benchmark criterion for the scrutiny of merger review that consumer-surplus-oriented competition authorities should adopt. I call the pair of technological synergies $(\hat{\Delta}^A, \hat{\Delta}^B)$ of strictly CS-neutral mergers as *CS-neutral technological synergies*.

Next, using the notion of CS-neutral technological synergies, I analyze the interaction between the firm's pre-merger network shares and the technological synergies required for strictly CS-neutral mergers. Unlike the mergers in one-sided markets, the relation between the types and the equilibrium market shares is not intuitively clear, and thus characterizing CS-neutral technological synergies in terms of the types of firms does not necessarily provide clear guidance. Thus, rather than directly characterizing CS-neutral technological synergies in terms of the types of merging parties, I use the pre-merger network shares of merging parties for characterizing the CS-neutral synergies. To this end, I take the following procedure:

1. First, for any fixed network shares n^A and n^B, compute the ι-markup implied by the first-order condition:
$$\mu^B = \frac{1}{1-n^B}\left(1-\beta\frac{n^A}{n^B}\right), \tag{36}$$
and
$$\mu^A = \frac{1}{1-n^A}. \tag{37}$$

2. Using these formulas for ι-markups, backup the types T^A and T^B using the formulas for network shares
$$n^A = \frac{T^A\left(T^B\right)^\beta}{H^A}\exp\left(-\mu^A-\beta\mu^B\right), \tag{38}$$
and
$$n^B = \frac{T^B}{H^B}\exp\left(-\mu^B\right). \tag{39}$$
Solving this system of equations with respect to T^A and T^B derives the type induced by network shares as $T^J = \tau^J(n^A, n^B, H^A, H^B)$ for $J = A, B$, where
$$\tau^A\left(n^A, n^B, H^A, H^B\right) := \frac{H^A}{\left(H^B\right)^\beta}\frac{n^A}{\left(n^B\right)^\beta}\exp\left(\frac{1}{1-n^A}\right), \tag{40}$$

and

$$\tau^B\left(n^A, n^B, H^A, H^B\right) := H^B n^B \exp\left(\frac{1}{1-n^B}\left(1-\beta\frac{n^A}{n^B}\right)\right). \tag{41}$$

Finally, with pre-merger network shares (n_f^A, n_f^B) and (n_g^A, n_g^B) and pre-merger aggregators (H^A, H^B), the type that achieves the strictly CS-neutral merger $\tau^J(n_f^A+n_g^A, n_f^B+n_g^B, H^A, H^B)$ for $J = A, B$ can be computed. Using this definition of the type, the CS-neutral technological synergies can be computed as functions

$$\begin{aligned}&\tilde{\Delta}^J\left(n_f^A, n_g^A, n_f^B, n_g^B, H^A, H^B\right)\\ :=&\tau^J\left(n_f^A+n_g^A, n_f^B+n_g^B, H^A, H^B\right)-\tau^J\left(n_f^A, n_f^B, H^A, H^B\right)-\tau^J\left(n_g^A, n_g^B, H^A, H^B\right).\end{aligned} \tag{42}$$

To understand the novel impact of two-sidedness on the competitive effects of mergers, I consider the mergers between similar firms where $n_f^J = n_g^J = n^J$ for $j = A, B$. Let $\theta := n^A/n^B$ be the ratio of network shares of side A to B. Then, the type for the strictly CS-neutral merger is given by

$$\tau^A(2n^A, 2n^B, H^A, H^B) = \frac{H^A}{H^B}2^{1-\beta}\frac{n^A}{\left(n^B\right)^\beta}\exp\left(\frac{1}{1-2n^A}\right) \tag{43}$$

and

$$\tau^B(2n^A, 2n^B, H^A, H^B) = H^B 2n^B \exp\left(\frac{1}{1-2n^B}(1-\beta\theta)\right) \tag{44}$$

The CS-neutral technological synergy on side J is positive if and only if

$$\frac{\tau^J(2n^A, 2n^B, H^A, H^B)}{2\tau^J(n^A, n^B, H^A, H^B)} > 1,$$

which is equivalent to the condition

$$\frac{1}{1-2n^A}-\frac{1}{1-n^A}-\beta\log 2 > 0 \tag{45}$$

for $J = A$ and

$$1-\beta\theta > 0 \tag{46}$$

for $J = B$, respectively. This fact directly leads to the following proposition.

Proposition 7. *Consider a merger between firms with the same pre-merger network shares n^A and n^B. Let $\theta := n^A/n^B$. There exists a critical value $\hat{n}^A$ such that CS-neutral technological synergy on side A is positive if and only if n^A is smaller than $\hat{n}^A$. CS-neutral technological synergy on side B is positive if and only if* $1-\beta\theta > 0$.

The fact that whether a strictly CS-neutral merger requires positive technological synergies only depends on the ratio of network shares θ is a striking feature of the two-sided markets. If the merging parties are sufficiently large on the subsidizing segment (θ being large), then the strictly CS-neutral merger does not involve any technological synergy on the subsidized segment, while it may still require technological synergy on the subsidizing segment, which is the segment where merging parties exercise market power. However, as in the case with direct network externalities, the benefit from the network expansion may offset the cost of the increase in markups accompanying mergers, which may renders even the technological synergies on side A unnecessary.

Note that in this setting, the condition $\beta\theta > 1$ is equivalent to the condition that merging parties set negative markups to consumers on the subsidized segment. In this sense, the CS-neutral technological synergy on subsidized segment is negative if and only if firms set negative markups before the merger. This can provide a practical test to evaluate the impact of two-sidedness on mergers involving platforms: consumers who are subsidized through negative markups are likely to benefit from mergers.

In summary, two-sidedness affects the welfare properties of mergers in two ways. First, for consumers on the subsidizing segment, a merger may be desirable when the benefit of network expansion outweighs the cost of the accompanying market power. Second, when the merging parties set negative markups to consumers on the subsidized segment, the merger increases the subsidization incentives and benefits consumers on the subsidized segment.

7. Discussion

The merger analysis in this study is based on a static framework of price competition with single-homing consumers. While it derives useful insights on the merger policy toward platforms, there are also several limitations due to the simplicity of the framework, such as the restriction on demand systems and the absence of consumer multi-homing, compatibility choices, dynamics of competition, and the endogenous mergers. This section discusses how these elements potentially alter the implication of the main analysis.

General demand system To adopt an aggregative-games approach, it is assumed that the demand system has an IIA property across firms' products. However, this restricts the substitution patterns among products and is not realistic assumption in several applications. To incorporate the rich substitution patterns into the merger analysis, one needs to give up exploiting aggregative properties and thus obtaining clear theoretical results. In this case, a better way to conduct merger analysis is resorting to the simulation with an estimated demand model, which is a standard procedure in the empirical industrial organization literature.

The main analysis in Section 3 also assumes that there is not outside option. This assumption can be relaxed in the following manner. Suppose that each consumer z has an outside option with value $\log H_0 + \varepsilon_{0z}$, where ε_{0z} follows an i.i.d. type-I extreme-value distribution, and chooses whether to participate in one of the networks. Then, the probability that a consumer participate in some network is characterized by the following equation:

$$n_f = \frac{\left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}}{H_0 \frac{\left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}}{(n_f)^{\alpha}} + H(p)}.$$

Then the network share can be written as the function $\tilde{n}_f(H_f, H)$. Since the demand of each product conditional on that consumers participate in some network is given by equation (8), the unconditional demand function is given by

$$-\tilde{n}_f(H_f(p_f), H(p))\frac{h_i'(p_i)}{H_f(p_f)}.$$

A similar analysis can be made with this demand system, and the cases where $H_0 = 0$ corresponds with the model in Section 3. Also, as α approaches to 0, the demand system corresponds with that of Nocke and Schutz (2018b).

Finally, I discuss the specification of network externalities. In the model, the form of indirect utility is given by equation (52), and thus network externalities enter into the utility in a logarithm form. While this specification is not common in the theoretical studies on competition in two-sided markets or competition with network externalities, which mainly use linear network externalities (Katz and Shapiro, 1985; Armstrong, 2006), logarithm specification of network externalities are often adopted in the empirical studies that try to estimate the magnitude of network externalities or consumers' preference for variety (Ohashi, 2003; Rysman, 2004, 2007). Whether linear or logarithm specification is more plausible depends on the application. In this study, I adopt the logarithm specification to generate a closed-form demand functions, which greatly improves the tractability of analysis. Another reason to adopt logarithm form of network externalities is that, under linear network externalities, even consumers' choice probability of each products given a price profile may not be unique, which necessarily complicates the analysis of price competition and merger policy.[11]

Multi-homing The model in the main analysis assumed that the consumers on both sides single-home, to describe the demand on each side as a function of single aggregator. However, in some environments, consumers on one side often multi-home, and consumers on both side multi-home in another environment. The framework of this study fails to capture the implication of such kind of multi-homing. Thus, when consumers are likely to multi-home, we need to modify the results of the analysis according to the environments under consideration.

Partly, the following type of ad-sponsored competitive-bottleneck model can be considered, which is a slight modification of Anderson and Peitz (2020):

$$v_i - A_i + \alpha \log n_f + \varepsilon_{iz}. \tag{47}$$

Then, the demand is given by

$$s_i = \frac{\exp(v_i - A_i)n_f^{\alpha}}{\sum_{f' \in \mathcal{F}} \sum_{j \in \mathcal{N}_{f'}^J} \exp(v_i - A_i)n_{f'}^{\alpha}} \tag{48}$$

On the advertisers' side, the inverse advertising demand is given by $r_i(A_i) = 1 + \frac{b_i}{A_i}$, which

[11]This issue is highlighted in Chapter 7.8 of Anderson, De Palma and Thisse (1992).

comes from a utility function $A_i + b_i \log A_i$ and generates the advertisers' surplus function as

$$S_i(A_i) = b_i + b_i \log A_i$$

Assuming that the marginal cost is zero, the profit of each firm is given by

$$\sum_{i \in \mathcal{N}_f} \frac{(H_f)^{\frac{1}{1-\alpha}}}{H} \frac{\exp(v_i - A_i)}{H_f} (A_i + b_i) . \tag{49}$$

The first-order condition for A_i is given by

$$\begin{aligned} &D_i - D_i (A_i + b_i) \\ &- \exp(v_i - A_i) \sum_{j \in \mathcal{N}_f} \left\{ \frac{\alpha}{1-\alpha} \frac{D_j}{H_f} \left(A_j + b_j\right) - \frac{(H_f)^{\frac{1}{1-\alpha}}}{H} \frac{1}{H_f} \frac{1}{1-\alpha} D_j \left(A_j + b_j\right) \right\} = 0, \end{aligned} \tag{50}$$

which can be simplified to

$$A_i + b_i = 1 + \frac{1}{1-\alpha} \Pi_f - \frac{\alpha}{1-\alpha} \frac{\Pi_f}{n_f} =: \mu_f \tag{51}$$

As a result, we have $\Pi_f = n_f \mu_f$. Finally, the type-aggregation property is preserved on the consumers' side.

One crucial difference from the model of single-homing consumers is that, in the competitive bottleneck framework, advertisers' surplus cannot be expressed by aggregators, which makes the welfare analysis on advertisers' side complicated.

Using this framework, we can show that any CS-neutral merger improves advertisers' welfare, which is partly in line with Anderson and Peitz (2020)'s "media see-saw" argument that there are several situations where the welfare effects on consumers and advertisers conflict with each other.

Compatibility choices The main analysis has assumed that once firms merge, their products immediately become compatible. However, the choice of compatibility itself is firms' choice variables, as discussed in Katz and Shapiro (1985). In this regard, the products owned by different firms can be compatible. When the compatibility is a choice variable, some policy implication might change. For example, when the strong network effects are likely to harm the consumer surplus, it may be more appropriate to require the compatibility between a product of the merged entity and the products of other firms.

Dynamic competition The model of this study assumed that firms compete in prices in a static manner. However, the digital markets are typically characterized by rapid technological changes because of R&D, continuous entries of startups, and the dynamic evolution of consumer bases. These dynamic considerations may make the merger analysis based on a static market share less useful to evaluate the competitive effects. One potential effect of dynamic consideration is that a dominant firm is probably more dominant in the future time period, which requires a more stringent merger policy than that under a static framework.

Endogenous merger Merger is endogenous in various aspects (Nocke and Whinston, 2010, 2013; Mermelstein, Nocke, Satterthwaite and Whinston, 2020). Firms merge only when it is profitable, which may have some dynamic implication (Nocke and Whinston, 2010). Similarly, an acquirer can choose a target to acquire, which is biased toward an increase in profit rather than an improvement in consumer welfare (Nocke and Whinston, 2013). Further, firms can choose whether to invest in an asset by themselves or to acquire a firm (Mermelstein et al., 2020). All of these aspects may affect the conclusion of the main analysis. The results of this study can be interpreted as a benchmark to consider more complicated situations.

8. Conclusion

This study used a model of multiproduct-firm oligopoly to analyze the impact of network externalities on the consumer-surplus effects of mergers. The impact of direct network externalities on the welfare properties of mergers depends on the sizes of merging parties relative to the industry. When an incumbent tries to acquire an innovative entrant, the incumbent may have a greater incentive to innovate due to the demand-side scale economies. In two-sided markets, the pre-merger structure of market shares on two sides of markets as well as the sizes of merging parties can predict the post-merger consumer surplus. These implications give theoretical guidance to the competition authorities involved in the merger policy toward platforms.

This study abstracts several aspects of the merger policy, which leaves the avenue for future research. First, the analytical framework is static and does not consider dynamics such as R&D competition and sequential mergers. Incorporating such dynamics would enrich some policy prescriptions. Second, the framework of this study focuses on the case where consumers on each side single-home. However, various online services, consumer behavior is better characterized by multi-homing. The possibility of multi-homing can affect platform competition and the potential effects of mergers on competition in important ways, as discussed by Anderson et al. (2019). It would be constructive to analyze the competitive effects of mergers in general settings with consumer multi-homing, but the discrete-choice framework of this study does not fit well to tackle with this issue. Thus, I leave these issues for future research.

A. Appendix A: General Framework of Multiproduct-Firm Oligopoly with Network Externalities

In this section, I present a general framework which our specific analyses belong to. There is an industry with two sides of markets A, B with a set $\mathcal{N}^J$ of imperfectly substitutable products on side $J \in \{A, B\}$, produced by a set of firms $\mathcal{F}$, with its generic element f. Each firm f produces the set $\mathcal{N}_f^J$ of products on side J, where $\mathcal{N}_f^J \cap \mathcal{N}_{f'}^J = \emptyset$ for $f \neq f'$ and $\bigcup_{f \in \mathcal{F}} \mathcal{N}_f^J = \mathcal{N}^J$.

A.1. Consumer Demand

There is a mass of consumers on side $J \in \{A, B\}$ who derive firm-level network externalities from the purchase of each product $i \in \mathcal{N}_f^J$, which depends on the number of consumers who purchase the products of firm f. Specifically, each consumer $z \in [0, 1]$ on side $J \in \{A, B\}$ yields the indirect subutility from the purchase of product $i \in \mathcal{N}_f^J$ by

$$\log h_i^J(p_i) + \alpha_J \log n_f^J + \beta_J \log n_f^I + \varepsilon_{iz}^J, \tag{52}$$

where $\log h_i^J(p_i)$ is the stand-alone indirect subutility from product i at price p_i, and n_f^J and n_f^I are the numbers of consumers on side J and $I \neq J$ who purchase products provided by firm f. $\alpha_J \in [0, 1)$ represents the magnitude of direct network externalities, $\beta_J \in [0, 1)$ represents the magnitude of indirect network externalities, and ε_{iz}^J is an idiosyncratic taste shock that follows i.i.d. type-I extreme value distributions. The direct and indirect network externalities are based on the firm-level number of participants of each network. I assume that network effects are not too strong so that $1 - \alpha_J - \beta_I > 0$ holds for each $J \in \{A, B\}$ and $I \neq J$, which guarantees that firms do not set infinite negative prices. An intuitive interpretation of this assumption is that a marginal increase in the stand-alone indirect utility of consumers on one side, 1, is greater than the accompanying increase in the indirect utility of consumers on two sides of markets from the resulting network expansions, $\alpha_J + \beta_I$. In the analysis, I adopt two specific forms of functions h_i^J. One is *MNL-class* demand specification where

$$h_i^J(p_i) = \exp\left(\frac{a_i - p_i}{\lambda^J}\right),$$

and another is *CES-class* demand specification where

$$h_i^J(p_i) = \begin{cases} a_i p_i^{1-\sigma^J} & \text{if } p_i > 0 \\ +\infty & \text{if } p_i \leq 0 \end{cases}$$

with $\sigma^J > 1$. In both specifications, a_i represents the quality of each product. I mean by the words “MNL-class" and “CES-class" that, if $\alpha_J = \beta_J = 0$, $J \in \{A, B\}$, the demand system obtained from indirect subutilities $h_i^J(p_i) = \exp\left(\frac{a_i - p_i}{\lambda^J}\right)$ and $h_i^J(p_i) = a_i p_i^{1-\sigma^J}$ corresponds with that of multinomial-logit and CES demand functions, respectively.

Given the network sizes $(n_f^A, n_f^B)_{f \in \mathcal{F}}$ and prices $p := (p_i)_{i \in \mathcal{N}^A \cup \mathcal{N}^B}$, consumers choose one product to purchase and the amount of the purchase. I assume that there is no outside option so that all consumers on side J purchase some product in the set $\mathcal{N}^J$. I further assume that consumers on each side single-home, that is, each consumer chooses only one product to purchase. From the above utility specification, the corresponding demand system is derived as a rational-expectation equilibrium among consumers. That is, based on the expectation over the network sizes, consumers choose their own decision to maximize the utilities, and the realized network sizes are consistent with the original expectation.

First, define the firm-level and industry-level aggregators as

$$H_f^J(p_f^J) = \sum_{i \in \mathcal{N}_f^J} h_i^J(p_i), \quad \text{where } p_f^J := (p_i)_{i \in \mathcal{N}_f^J}.$$

$$H^J(p) = \sum_{f \in \mathcal{F}} \left(H_f^J(p_f^J)\right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I(p_f^I)\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}.$$

Next, I derive demand for each product conditional on the purchase. Applying Roy's identity, the conditional demand function for product i conditional on the purchase is given by $-(h_i^J)'(p_i)/h_i^J(p_i)$. I assume that consumers form the correct expectation that all firm have positive network shares. I call the network choice of consumers based on such expectation as an *interior consumption equilibrium*. Applying Holman and Marley's Theorem, the consumer choice probability s_i^J of product $i \in \mathcal{N}_f^J$ given the expectation over network shares $(n_{f'}^J)_{f' \in \mathcal{F}}$, $J = A, B$ is given by

$$s_i^J = \frac{h_i^J(p_i)\left(n_f^J\right)^{\alpha_J}\left(n_f^I\right)^{\beta_J}}{\sum_{f' \in \mathcal{F}} \sum_{j \in \mathcal{N}_{f'}^J} h_j^J(p_j)\left(n_{f'}^J\right)^{\alpha_J}\left(n_{f'}^I\right)^{\beta_J}}. \tag{53}$$

I require the network share is consistent with the consumers' behaviors, that is, the network share n_f^J of firm f on side J is given by the sum of the choice probability of products produced by firm f on side J:

$$n_f^J = \sum_{i \in \mathcal{N}_f^J} s_i^J. \tag{54}$$

From equations (53) and (54), the share of product $i \in \mathcal{N}_f^J$ in the set of products sold by firm f is given by

$$\frac{s_i^J}{n_f^J} = \frac{h_i^J(p_i)}{H_f^J(p_f^J)}. \tag{55}$$

As derived in the Appendix A.5, the network share n_f^J of firm f on side J in the interior consumption equilibrium is given by

$$n_f^J(p) = \frac{1}{H^J(p)}\left(\left((H_f^J(p_f^J)\right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}}\left(H_f^I(p_f^I)\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}\right). \tag{56}$$

Combining equations (55) and (56) the probability that product $i \in \mathcal{N}_f^J$ is purchased by a consumer is given by the equation

$$s_i^J(p) = n_f^J(p)\frac{h_i^J(p_i)}{H_f^J(p_f^J)}. \tag{57}$$

Finally, the demand for the product $i \in \mathcal{N}_f^J$ given the profile of prices p has the following form.

$$
\begin{aligned}
D_i^J(p) &= \hat{D}_i^J\left(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p)\right) \\
&= s_i^J(p) \times \frac{-(h_i^J)'(p_i)}{h_i^J(p_i)} \\
&= -\left(H_f^J(p_f^J)\right)^{\frac{(1-\alpha_I)\alpha_J+\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I(p_f^I)\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}} \frac{(h_i^J)'(p_i)}{H^J(p)}
\end{aligned}
\tag{58}
$$

With CES-class demand and negative price, we cannot use Roy's identity to derive demand. To allow for the demand for negative prices, we assume that $D_i^J(p) = \lim_{p_i \to 0} D_i^J(p) = +\infty$ for $p_i < 0$.

Finally, the consumer surplus CS^J on side J is given by the expected indirect utility of consumers, and the aggregate consumer surplus CS is given by the some of consumer surplus on both sides :

$$
\begin{aligned}
CS^J &= \log\left(\sum_{f\in\mathcal{F}} \left(H_f^J\right)^{\frac{1-\alpha_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)\beta_J\beta_I}} \frac{1}{(H^J)^{\alpha_J}(H^I)^{\beta_J}}\right) \\
&= (1-\alpha_J)\log H^J - \beta_J \log H^I,
\end{aligned}
\tag{59}
$$

and

$$
\begin{aligned}
CS &= CS^A + CS^B \\
&= (1-\alpha_A-\beta_B)\log H^A + (1-\alpha_B-\beta_A)\log H^B.
\end{aligned}
\tag{60}
$$

Note that in the presence of indirect network externalities $\beta_J > 0$, the consumer surplus on side J is decreasing in the value of aggregator H^I on side $I \neq J$ while it increases with the value of the aggregator on the same side H^J. This is because the large value of aggregator on the other side shrinks the market share and thereby resulting in the fragmentation of networks.

A.2. Firm pricing

Each product $i \in \mathcal{N}^J$ has a constant marginal cost $c_i > 0$ of production. Given the demand system, the profit function of each firm $f \in \mathcal{F}$ is written as a function of the profile of firm's own prices $p_f = (p_i)_{\mathcal{N}_f^A \cup \mathcal{N}_f^B}$ and aggregators H^A and H^B:

$$
\Pi_f\left(p_f, H^A(p), H^B(p)\right) = \Pi_f^A + \Pi_f^B, \tag{61}
$$

where

$$
\Pi_f^J = \sum_{i\in\mathcal{N}_f^J} \hat{D}_i^J\left(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p)\right)(p_i - c_i). \tag{62}
$$

The pricing game consists of a demand system $\{(D_i^J)_{i\in\mathcal{N}^J}\}_{J\in\{A,B\}}$, the set of firms $\mathcal{F}$, and a profile of marginal costs $(c_i)_{i\in\mathcal{N}^J}$, $J \in \{A, B\}$. In a pricing game, firms simultaneously set the prices $p_f := (p_i)_{i \in \mathcal{N}_f^A \cup \mathcal{N}_f^B}$ of their products, with the payoff function Π_f defined by equation (61). I call a Nash equilibrium of this pricing game as a *pricing equilibrium*. In the following analysis, I often suppress the arguments of functions for the sake of readability.

As formally derived in Appendix A.5, the first-order condition for the profit-maximization of each firm f is given by

$$
\begin{aligned}
&-\frac{(h_i^J)''}{(h_i^J)'}(p_i - c_i) \\
&= 1 - \underbrace{\frac{1}{n_f^J}\frac{\{(1-\alpha_I)\alpha_J + \beta_J\beta_I\}\Pi_f^J + \beta_I\Pi_f^I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}}_{\text{network-externality terms}} + \underbrace{\frac{(1-\alpha_I)}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\left(\Pi_f^J + \frac{\beta_I}{1-\alpha_I}\frac{n_f^I}{n_f^J}\Pi_f^I\right)}_{\text{cannibalization terms}} \\
&=: \mu_f^J
\end{aligned}
\tag{63}
$$

As Nocke and Schutz (2018b) do, I call μ_f^J as the ι-markup of firm f on side J. This ι-markup summarizes the pricing incentive of each firm.

Let me explain how the ι-markup of each firm is determined in the equation (63). the first term, 1, in the second line of the equation (63) is the baseline ι-markup, which would be set under the monopolistic competition. The second term is the downward-pricing pressure due to the direct and indirect network externalities. The third term is the upward-pricing pressure due to the cannibalization effects under oligopoly. The relative magnitudes of the second and the last terms on each side determine the price level and the price structure of each firm.

We have $-(h_i^J)''/(h_i^J)' = 1/\lambda^J$ and thus $p_i = c_i + \lambda^J \mu_f^J$ in the case of multinomial logit demand, and $-(h_i^J)''/(h_i^J)' = \sigma^J/p_i$ and thus $p_i = c_i/(1 - \mu_f^J/\sigma^J)$ in the case of CES-class demand. Using these functional forms, the formula for the subaggregators and profit functions are given by

$$
H_f^J = \begin{cases} T_f^J \exp(-\mu_f^J) & \text{in the case of MNL-class demand,} \\ T_f^J \left(1 - \frac{\mu_f^J}{\sigma^J}\right)^{\sigma^J - 1} & \text{in the case of CES-class demand,} \end{cases} \tag{64}
$$

and

$$
\Pi_f^J = \begin{cases} n_f^J \mu_f^J & \text{in the case of MNL-class demand,} \\ \frac{\sigma^J - 1}{\sigma^J} n_f^J \mu_f^J & \text{in the case of CES-class demand,} \end{cases} \tag{65}
$$

where $T_f^J := \sum_{i \in \mathcal{N}_f^J} \exp\left(\frac{a_i - c_i}{\lambda^J}\right)$ for MNL-class and $T_f^J := \sum_{i \in \mathcal{N}_f^J} a_i c_i^{1-\sigma^J}$ for CES-class demand. T_f^J is the "type" of firm f that corresponds with the value of the subaggregator of firm f when it engages in the marginal cost pricing. The property that all the pricing information is summarized by unidimensional type T_f^J is called as the "type-aggregation property" (Nocke and Schutz, 2018b). This property greatly simplifies the analysis of merger policy. As a result, the ι-markup μ_f^J and the network share n_f^J depends only on T_f^A, T_f^B, H^A, and H^B. Whenever they are unique, I write the ι-markups μ_f^J and market shares n_f^J under each firm's optimal pricing as functions

$$\mu_f^J = m^J\left(T_f^J, T_f^I, H^J, H^I\right), \tag{66}$$

and

$$n_f^J = N^J\left(T_f^J, T_f^I, H^J, H^I\right) \tag{67}$$

for $J \in \{A, B\}$.

A.3. Pricing Equilibrium

Given the above market share functions, the equilibrium condition for the aggregators (H^A, H^B) is

$$\sum_{f \in \mathcal{N}_f^J} N^J\left(T_f^J, T_f^I, H^J, H^I\right) = 1, \quad J \in \{A, B\}, \quad I \neq J. \tag{68}$$

The general characterization of the equilibrium and the analysis of merger policy are not easy to conduct. Thus, to obtain the clear-cut insights, I divide the analysis of merger policy into two special cases where there is only direct network externalities and where there is only indirect network externalities. For each special case, I separately prove the existence and the uniqueness of the equilibrium.

Proposition 8. *(Uniqueness of equilibrium) The following statements hold:*

1. *For any pair of aggregators (H^A, H^B), each firm's optimal pricing is characterized by the first-order condition (63).*
2. *If $\beta_A = \beta_B = 0$, then there is unique pricing equilibrium.*
3. *If the demand system is given by MNL-class demand, $\alpha_A = \alpha_B = 0$, and either β_A or β_B is close to 0, then there is unique pricing equilibrium.*

I provide the proof of this proposition in Appendix A.4.

The first part of Proposition 8 shows that in any equilibrium, each firm's pricing is characterized by the first-order condition (63). The second and the third parts of Proposition 8 guarantee the existence and the uniqueness of equilibrium in special cases analyzed in Section 4. and Section 6.

A.4. Proof of Proposition 8

A.4.1. Proof of Proposition 8.1

I prove Proposition 1-1 for CES-class demand and MNL-class demand separately.

Before proceeding to individual proofs, I introduce several notations that are used in both proofs. First, let

$$\Omega_f^{JI}(p_f) := \left(H_f^J(p_f^J)\right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I(p_f^I)\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}}$$

and

$$H^J_{-f} = \sum_{f' \in \mathcal{F} \setminus \{f\}} \Omega^{JI}_{f'}(p_{f'}).$$

Then, the profit-maximization problem of firm f the problem can be rewritten as

$$\max_{p_f \in \mathbb{R}^{\mathcal{N}^A_f \cup \mathcal{N}^B_f}} G_f(p_f) := \Pi_f \left(p_f, \Omega^{AB}_f(p_f) + H^A_{-f}, \Omega^{BA}_f(p_f) + H^B_{-f} \right) \tag{69}$$

I show that the solution to (69) takes unique finite value and given by the first-order condition (63). Here, I list the steps of the proof. For CES-class demand,

1. To whatever extent a subaggregator H^J_f of firm f on one side grows, the profit on the other side Π^I_f is bounded above.
2. Setting zero price for some good is never optimal.
3. Setting infinite price for some good is never optimal.
4. The optimal prices should satisfy the first-order condition (63).

For MNL-class demand,

1. Fixing p^A_f, p^B_f that maximizes $G_f(p^A_f, p^B_f)$ is finite and unique. Let $\tilde{p}^B_f(p^A_f)$ denote such p^B_f.
2. p^A_f that maximizes $G_f(p^A_f, \tilde{p}^B_f(p^A_f))$ is finite and unique.
3. Setting infinite price for some good is never optimal.
4. The optimal prices should satisfy the first-order condition (63).

CES-class demand I first show that whatever the value of subaggregator H^J_f is, the value Π^I_f is bounded. To see this, note that

$$\begin{aligned}\Pi^I_f = \frac{\Omega^{IJ}}{H^I} \sum_{j \in \mathcal{N}^I_f} \frac{-(h^I_j)'(p_j)}{H^I_f(p^I_f)}(p_j - c_j) \leq \sum_{j \in \mathcal{N}^I_f} \frac{-(h^I_j)'(p_j)}{H^I_f(p^I_f)}(p_j - c_j) &= \frac{\sum_{j \in \mathcal{N}^I_f (\sigma_J - 1)(a_j p_j^{-\sigma_I})(p_j - c_j)}}{\sum_{k \in \mathcal{N}^I_f a_k p_k^{\sigma_I - 1}}} \\ &\leq \sigma_J - 1,\end{aligned} \tag{70}$$

where the last inequality follows from computing $\max_{p^I_f} \left\{ \sum_{j \in \mathcal{N}^I_f (\sigma_J - 1)(a_j p_j^{-\sigma_I})(p_j - c_j)} \right\} / \left\{ \sum_{k \in \mathcal{N}^I_f a_k p_k^{\sigma_I - 1}} \right\}$

Next, I show that the price of each product should be in the set $(0, \infty)$, and thus the optimal pricing should be characterized by the equation (63). To see this, suppose that $p_i \leq 0$ for some $i \in \mathcal{N}^J$. Then, $h^J_i(p_i) = \infty$ and thus the $D^J_j(p) = 0$ for all $j \in \mathcal{N}^J_f \setminus \{i\}$. Therefore the profit of firm f is given by

$$D^J_i(p)(p_i - c_i) + \Pi^I_f. \tag{71}$$

Since Π_f^I is finite as shown above, $D_i^J(p) = \infty$, and $p_i - c_i < 0$, the profit is negative. Thus, setting non-positive for some product is never optimal for firm f.

The fact that firm never sets infinite price is shown in the same manner as Nocke and Schutz (2018a).

Thus, the optimal profile of prices is interior, which implies that the optimal prices should satisfy the first-order condition (63).

MNL-class demand I first show that all firms' prices are bounded below. Next, I show that all firm' prices are bounded above.

Fix $p_f^I := (p_i^I)_{i \in \mathcal{N}_f^I}$. Then, I show that the value of $(p_f^J) := (p_i^J)_{i \in \mathcal{N}_f^J}$ that maximize the profit of firm f has finite absolute values. To see this, note that

$$\begin{aligned}&\operatorname{sign}\left(\frac{\partial G(p_f)}{\partial p_i}\right)\\ =&\operatorname{sign}\left(1 - \frac{p_i - c_i}{\lambda_J} + \frac{(1-\alpha_I) - \{(1-\alpha_I)\alpha_J + \beta_I\beta_J\}\frac{1}{n_f^J}}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\Pi_f^J - \frac{\beta_I}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\frac{(1-n_f^I)}{n_f^J}\Pi_f^I\right).\end{aligned} \tag{72}$$

the last term converges to 0 as $p_i \to -\infty$ because $n_f^I \to 1$ as $p_i \to -\infty$. The sum of the second and the third terms is nonnegative as $p_i \to -\infty$ because

$$\begin{aligned}&-\frac{p_i - c_i}{\lambda_J} + \frac{(1-\alpha_I) - \{(1-\alpha_I)\alpha_J + \beta_I\beta_J\}\frac{1}{n_f^J}}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\Pi_f^J\\ &\geq -\frac{p_i - c_i}{\lambda_J} + \Pi_f^J\\ &\geq -\frac{p_i - c_i}{\lambda_J} + n_f^J\frac{p_i - c_i}{\lambda_J}\end{aligned} \tag{73}$$

as $p_i \to -\infty$. Thus we have $\partial G/\partial p_i > 0$ for sufficiently small p_i.

The fact that firm never sets infinite price is shown in the same manner as Nocke and Schutz (2018a).

As a result, fixed the values of p_f^I, the optimal pricing for p_f^J is given by the common ι-markup pricing, which is given by (63). Let $\mu_f^J(p_f^I)$ be the optimal ι-markup on side J given the profile of prices on the other side p_f^I. Then, under MNL-class demand, we have $p_i = c_i + \lambda_J\mu_f^J(p_f^I)$ for $i \in \mathcal{N}_f^J$.

Next, I show that the optimal value of p_f^I that maximizes $G(p)$ where $p_f^J = (c_i + \lambda_J\mu_f^J(p_f^I))_{i \in \mathcal{N}_f^J}$ is finite. To see this, it is sufficient to show that

$$\begin{aligned}
&\text{sign}\left(\frac{\partial G((p^I_f, (c_i + \lambda_J \mu^J_f(p^I_f))_{i\in\mathcal{N}^J_f})}{\partial p_j}\right)\\
=&\text{sign}\left(1 - \frac{p_j - c_j}{\lambda_I} + \frac{(1-\alpha_J) - \{(1-\alpha_I)\alpha_J + \beta_I\beta_J\}\frac{1}{n^I_f}}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\Pi^I_f - \frac{\beta_J}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\frac{(1-n^J_f)}{n^I_f}\Pi^J_f\right).
\end{aligned} \tag{74}$$

Using the first-order condition for μ^J_f, 63, we have

$$\begin{aligned}
&\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}(1-n^J_f)\mu^J_f\\
=&1 - \frac{\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\frac{n^I_f}{n^J_f}(1-n^I_f)\sum_{j'\in\mathcal{N}^I_f}\frac{\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{H^I_f}\frac{p_{j'}-c_{j'}}{\lambda^I}
\end{aligned} \tag{75}$$

By l'Hopital's rule, we have

$$\begin{aligned}
&\lim_{p_i\to-\infty}\frac{\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\frac{n^I_f}{n^J_f}(1-n^I_f)\sum_{j'\in\mathcal{N}^I_f}\frac{\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{H^I_f}\frac{p_{j'}-c_{j'}}{\lambda^I}\\
=&\lim_{p_j\to-\infty}\frac{\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\frac{\sum_{j'\in\mathcal{N}^I_f\setminus\{i\}}\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{\sum_{j'\in\mathcal{N}^I_f\setminus\{i\}}\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right) + \exp\left(\frac{a_i-p_i}{\lambda^I}\right)}\frac{p_i-c_i}{\lambda^I}\\
=&\lim_{p_j\to-\infty}\frac{\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\frac{\sum_{j'\in\mathcal{N}^I_f\setminus\{i\}}\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{-\lim_{p_i\to\infty}\lambda^I\exp\left(\frac{a_i-p_i}{\lambda^I}\right)}\\
=&0.
\end{aligned}$$

Thus, we have

$$\lim_{p_j\to-\infty}\frac{(1-n^J_f)}{n^I_f}\Pi^J_f = \frac{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}{1-\alpha_I},$$

and

$$\begin{aligned}
&\text{sign}\lim_{p_j\to-\infty}\left(\frac{\partial G((p^I_f, (c_i + \lambda_J \mu^J_f(p^I_f))_{i\in\mathcal{N}^J_f})}{\partial p_j}\right)\\
=&\text{sign}\lim_{p_j\to-\infty}\left(1 - \frac{\beta_J}{1-\alpha_I} - \frac{p_j-c_j}{\lambda_I} + \frac{(1-\alpha_J) - \{(1-\alpha_I)\alpha_J + \beta_I\beta_J\}\frac{1}{n^I_f}}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\Pi^I_f\right) > 0.
\end{aligned} \tag{76}$$

as $p_j \to -\infty$ for some $\mathcal{N}_f^I$.

Again, the fact that firm never sets infinite price is shown in the same manner as Nocke and Schutz (2018a).

Thus, the optimal profile of prices is interior, which implies that the optimal prices should satisfy the first-order condition (63).

Note that in when $\beta_A = \beta_B = 0$ or $\alpha_A = \alpha_B = 0$ and either $\beta_A \simeq 0$ or $\beta_B \simeq 0$, equation (63) is also sufficient for the profit maximization. In the subsequence sections (Appendix A.4.2-A.4.3), I prove that the solution to (63) is unique when $\beta_A = \beta_B = 0$ or $\alpha_A = \alpha_B = 0$ and either $\beta_A \simeq 0$ or $\beta_B \simeq 0$. Then, by the facts that (i) there is finite price profile that maximizes the profit, (ii) any optimal price profile should satisfy equation (63), and (iii) when $\beta_A = \beta_B = 0$ or $\alpha_A = \alpha_B = 0$ and either $\beta_A \simeq 0$ or $\beta_B \simeq 0$, the pair of ι-markups (μ_f^A, μ_f^B) that satisfies equation (63) is unique jointly show that there is a unique solution to (63) that maximizes the firm's profit. If the solution to (63) does not maximize the firm's profit, then there must be some finite price profile that maximizes the firm's profit but does not satisfy equation (63), which contradicts the necessity of (63).

A.4.2. Proof of Proposition 8.2

Suppose that $\beta_A = \beta_B = 0$. Then, two sides of markets are independent, and thus it suffice to focus on one side of the market. The ι-markup is uniquely given by the equation (FOC-MNL) for MNL-class demand and (FOC-CES) for CES-class demand. Finally, since the market share equation $N_0(\gamma(T)/H)$ defined by is monotonically decreasing in H, $\lim_{x\to 0} N_0(x) = 0$, and $\lim_{x\to\infty} N_0(x) = 1$, the intermediate value theorem implies that the value of aggregator H that satisfies $\sum_f N_0(\gamma(T_f)/H) = 1$ is unique.

A.4.3. Proof of Proposition 8.3

I show that there is the unique pair of ι-markup that satisfies the system of equations (63), and the unique pair of aggregators that satisfies the system of equation (63) when the demand system is given by MNL-class demand, $\alpha_A = \alpha_B = 0$, and at at least one of β_A or β_B is sufficiently close to 0.

Because the solution to equation (63) is continuous in β_J at $\beta_J = 0$, without loss of generality, suppose that $\beta_B = 0$. After several manipulations, the system of first-order conditions (63) can be rewritten as

$$g_A(\mu_f^A, \mu_f^B) = (1 - n_f^A)\mu_f^A - 1 = 0, \tag{77}$$

$$g_B(\mu_f^A, \mu_f^B) = (1 - n_f^B)\mu_f^B - 1 + \beta_A \frac{n_f^A}{n_f^B} = 0. \tag{78}$$

Let $g(\mu_f^A, \mu_f^B) := \{g_A(\mu_f^A, \mu_f^B, g_B(\mu_f^A, \mu_f^B)\}$. To show that this system of equations has unique solution, I show that the determinant of $D(g(\mu_f^A, \mu_f^B)$ is positive.[12] A calculation leads to

$$\begin{aligned}\det G_f =& \left(\frac{n_f^A}{1-n_f^A} + 1 - n_f^A\right)\left(\frac{n_f^B}{1-n_f^B} + 1 - n_f^B + \beta_A\left(\frac{1}{n_f^B} - \frac{1}{1-n_f^B}\right)\right) \\ &+ \beta_A^3 \frac{(n_f^A)^2}{n_f^B(1-n_f^A)} > 0,\end{aligned} \tag{79}$$

where

$$G_f := \begin{pmatrix} \frac{\partial g_A}{\partial \mu_f^A} & \frac{\partial g_A}{\partial \mu_f^B} \\ \frac{\partial g_B}{\partial \mu_f^A} & \frac{\partial g_B}{\partial \mu_f^B} \end{pmatrix}, \tag{80}$$

and the last inequality follows from the fact that

$$\begin{aligned}&\frac{n_f^B}{1-n_f^B} + 1 - n_f^B + \beta_A\left(\frac{1}{n_f^B} - \frac{1}{1-n_f^B}\right) \\ =& \frac{1}{n_f^B(1-n_f^B)}\left((n_f^B)^3 - (n_f^B)^2 + n_f^B + \beta_A n_f^A(1-2n_f^B)\right)\end{aligned} \tag{81}$$

is positive. To see this, all the terms in expression (81) is positive when $n_f^B < 1/2$. When $n_f^B \geq 1/2$, the last term in expression (81) is nonpositive. In that case, by the fact that $\beta_A n_f^A \in (0,1)$ the following inequality holds:

$$\nu(n_f^B) := (n_f^B)^3 - (n_f^B)^2 + n_f^B + \beta_A n_f^A(1-2n_f^B) \geq (n_f^B)^3 - (n_f^B)^2 - n_f^B + 1, \tag{82}$$

which is positive for all $n_f^B \in (0,1)$. This is because $\nu(1) = 0$, and $\nu'(n_f^B) < 0$ for all $n_f^B \in (0,1)$.

Thus, the pair of ι-markups that satisfies the first-order condition (63) is unique.

Finally, I show that there is unique equilibrium. To do this, I present several comparative statics of m^J, $J \in \{A, B\}$, with respect to several parameters x. This is given by the Implicit Function Theorem

$$G_f \begin{pmatrix} \frac{\partial m^A}{\partial x} \\ \frac{\partial m^B}{\partial x} \end{pmatrix} = -\begin{pmatrix} \frac{\partial g_A}{\partial x} \\ \frac{\partial g_B}{\partial x} \end{pmatrix}. \tag{83}$$

by Cramer's Rule, I obtain

$$\frac{\partial \mu^A}{\partial x} = \frac{\det\begin{pmatrix} -\frac{\partial g_A}{\partial x} & \frac{\partial g_A}{\partial \mu_B} \\ -\frac{\partial g_B}{\partial x} & \frac{\partial g_B}{\partial \mu_B} \end{pmatrix}}{\det G_f}, \quad \frac{\partial \mu^B}{\partial x} = \frac{\det\begin{pmatrix} \frac{\partial g_A}{\partial \mu_A} & -\frac{\partial g_A}{\partial x} \\ \frac{\partial g_B}{\partial \mu_A} & -\frac{\partial g_B}{\partial x} \end{pmatrix}}{\det G_f}. \tag{84}$$

Using this comparative statics in ι-markups, I conduct a comparative statics in market shares

[12] See chapter 2 of Vives (2001).

N^A and N^B:

$$\frac{\partial N^A}{\partial x} = \frac{\partial n^A}{\partial x} - \frac{\partial m^A}{\partial x} n^A - \beta_A \frac{\partial m^B}{\partial x} n^A \tag{85}$$

$$\frac{\partial N^B}{\partial x} = \frac{\partial n^B}{\partial x} - \frac{\partial m^B}{\partial x} n^B \tag{86}$$

Based on this observation, I derive the effects of H^A and H^B on N^A and N^B.

Fist, for H^A, we have

$$\begin{aligned}
\frac{\partial m^A}{\partial H^A} &= -\frac{n_f^A}{H^A} \frac{1}{\det(G_f)} \left[\mu_f^A \left(n_f^B \mu_f^B + 1 - n_f^B + \beta_A \frac{n_f^A}{n_f^B} \right) + \mu_f^A \beta_A^2 \frac{n_f^A}{n_f^B} \right] < 0 \\
\frac{\partial m^B}{\partial H^A} &= \frac{n_f^A}{H^A} \frac{1}{\det(G_f)} \left(\beta_A \frac{n_f^A}{n_f^B} \mu_f^A (1 - \beta_A) + \beta_A \frac{1 - n_f^A}{n_f^B} \right) > 0,
\end{aligned}$$

and thus

$$\frac{\partial N^A}{\partial H^A} = -\frac{n_f^A}{H^A} \frac{1}{\det(G_f)} (1 - n_f^A) \left(n_f^B \mu_f^B + 1 - n_f^B + \beta_A (1 + \beta_A) \frac{n_f^A}{n_f^B} \right) < 0 \tag{87}$$

$$\frac{\partial N^B}{\partial H^A} = -\frac{n_f^A}{H^A} \frac{1}{\det(G_f)} \left(\beta_A \frac{n_f^A}{n_f^B} \mu_f^A (1 - \beta_A) + \beta_A \frac{1 - n_f^A}{n_f^B} \right) n_f^B < 0. \tag{88}$$

For H^B, we have

$$\begin{aligned}
\frac{\partial m^A}{\partial H^B} &= \frac{n_f^B}{H^B} \frac{1}{\det(G_f)} \left(\mu_f^B + \beta_A \frac{n_f^A}{(n_f^B)^2} \right) \beta_A n_f^A \mu_f^A > 0 \\
\frac{\partial m^B}{\partial H^B} &= -\frac{n_f^B}{H^B} \frac{1}{\det(G_f)} \left(\mu_f^B + \beta_A \frac{n_f^A}{(n_f^B)^2} \right) (n_f^A \mu_f^A + 1 - n_f^A) < 0,
\end{aligned}$$

and thus

$$\frac{\partial N^A}{\partial H^B} = \beta_A \frac{n_f^A}{n_f^B} \frac{n_f^B}{H^B} \frac{1}{\det(G_f)} \left(n_f^B \mu_f^B + \beta_A \frac{n_f^A}{n_f^B} \right) \left(1 - n_f^A \right) > 0, \tag{89}$$

$$\frac{\partial N^B}{\partial H^B} = -\frac{n_f^B}{H_f^B} \frac{1}{\det(G_f)} \left((n_f^A \mu_f^A + 1 - n_f^A)(1 - n_f^B) + \beta_A^3 \frac{n_f^A}{n_f^B} n_f^A \mu_f^A \right) < 0. \tag{90}$$

As a result, we have

$$\det \begin{pmatrix} \sum \frac{\partial N^A}{\partial H^A} & \sum \frac{\partial N^A}{\partial H^B} \\ \sum \frac{\partial N^B}{\partial H^A} & \sum \frac{\partial N^B}{\partial H^B} \end{pmatrix} > 0, \tag{91}$$

which implies that the pair (H^A, H^B) that satisfies the condition (68) is unique.

A.5. Omitted derivation of demand system and firm pricing in Section A.1 and A.2

Consumer demand I first present a general framework to which our specific analyses belong to.

$$\log h_i^J(p_i) + \alpha_J \log n_f^J + \beta_J \log n_f^I + \varepsilon_i, \tag{92}$$

where $J, I \in \{A, B\}$, $J \neq I$, $\alpha^J \in [0, 1)$, and ε follows the Type-I extreme-value distribution. As a result, the consumer choice probability of product $i \in \mathcal{N}_f^J$ is given by

$$s_i^J = \frac{h_i^J(p_i)(n_f^J)^{\alpha_J}(n_f^I)^{\beta_J}}{\sum_{f' \in \mathcal{F}} \sum_{j \in \mathcal{N}_f^J} h_j^J(p_j)(n_{f'}^J)^{\alpha_J}(n_{f'}^I)^{\beta_J}} \tag{93}$$

network share n_f^J is given by

$$n_f^J = \frac{H_f^J(n_f^J)^{\alpha_J}(n_f^I)^{\beta_J}}{\sum_{f' \in \mathcal{F}} H_{f'}^J(n_{f'}^J)^{\alpha_J}(n_{f'}^I)^{\beta_J}}. \tag{94}$$

Then, we have

$$\frac{n_f^J}{n_{f'}^J} = \frac{H_f^J}{H_{f'}^J} \frac{(n_f^J)^{\alpha_J}}{(n_{f'}^J)^{\alpha_J}} \frac{(n_f^I)^{\beta_J}}{(n_{f'}^I)^{\beta_J}} \tag{95}$$

which can be rewritten as

$$\frac{n_f^J}{n_{f'}^J} = \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1}{1-\alpha_J}} \left(\frac{n_f^I}{n_{f'}^I}\right)^{\frac{\beta_J}{1-\alpha_J}} = \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1}{1-\alpha_J}} \left(\left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{1}{1-\alpha_I}} \left(\frac{n_f^J}{n_{f'}^J}\right)^{\frac{\beta_I}{1-\alpha_I}}\right)^{\frac{\beta_J}{1-\alpha_J}}. \tag{96}$$

Finally, we obtain

$$\frac{n_f^J}{n_{f'}^J} = \left(\left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1}{1-\alpha_J}} \left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{\beta_J}{(1-\alpha_I)(1-\alpha_J)}}\right)^{\frac{1}{1-\frac{\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)}}} = \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \tag{97}$$

Plugging this equation into the definition of market share function, I obtain the closed-form expression for the network share

$$
\begin{aligned}
n_f^J &= \frac{H_f^J}{\sum_{f'\in\mathcal{F}} H_{f'}^J \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{(1-\alpha_I)\alpha_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{\alpha_J\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_{f'}^I}{H_f^I}\right)^{\frac{(1-\alpha_J)\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_{f'}^J}{H_f^J}\right)^{\frac{\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}}} \\
&= \frac{(H_f^J)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_f^I)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}}{\sum_{f'\in\mathcal{F}} (H_{f'}^J)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_{f'}^I)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}}
\end{aligned}
\tag{98}
$$

and the demand for product $i \in \mathcal{N}_f^J$

$$
\begin{aligned}
D_i^J(p) &= \hat{D}_i^J(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p)) \\
&= n_f^J(p) \times \frac{s_i^J(p)}{n_f^J(p)} \times \left(-\frac{(h_i^J)'(p_i)}{h_i^J(p_i)}\right) \\
&= -\frac{(H_f^J(p_f^J))^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_f^I(p_f^I))^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}}{H^J(p)} \frac{(h_i^J)'(p_i)}{H_f^J(p_f^J)} \\
&= -\frac{(H_f^J(p_f^J))^{\frac{(1-\alpha_I)\alpha_J+\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_f^I(p_f^I))^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}}{H^J(p)} (h_i^J)'(p_i)
\end{aligned}
\tag{99}
$$

Firm pricing Given this demand system derived above, the profit function of each firm f is given by

$$
\begin{aligned}
\Pi_f(p_f, H^A(p), H^B(p)) =& \Pi_f^A(p_f^A, H_f^A(p_f^A), H_f^B(p_f^B), H^A(p), H^B(p)) \\
&+ \Pi_f^B(p_f^B, H_f^B(p_f^B), H_f^A(p_f^A), H^B(p), H^A(p))
\end{aligned}
\tag{100}
$$

where

$$
\Pi_f^J(p_f^J, H_f^J(p_f^J), H_f^I(p_f^I)) = \sum_{i\in\mathcal{N}_f^J} \hat{D}_i^J(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p))(p_i - c_i). \tag{101}
$$

The first-order condition for profit-maximization of each firm f is given by

$$\begin{aligned}
0 &= \frac{\partial \Pi_f^J}{\partial p_i} + (h_i^J)' \left(\frac{\partial \Pi_f^J}{\partial H_f^J} + \frac{\partial \Pi_f^I}{\partial H_f^J} + \frac{\partial H^J}{\partial H_f^J} \frac{\partial \Pi_f^J}{\partial H^J} + \frac{\partial H^I}{\partial H_f^J} \frac{\partial \Pi_f^I}{\partial H^I} \right) \\
&= \hat{D}_i^J - \hat{D}_i^J \frac{(h_i^J)''}{(h_i^J)'}(p_i - c_i) + (h_i^J)' \left(\frac{(1-\alpha_I)\alpha_J + \beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I} \frac{1}{H_f^J} \Pi_f^J + \frac{\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I} \frac{1}{H_f^J} \Pi_f^I \right) \\
&\quad - (h_i^J)' \left(\frac{\partial H^J}{\partial H_f^J} \frac{1}{H^J} \Pi_f^J + \frac{\partial H^I}{\partial H_f^J} \frac{1}{H^I} \Pi_f^I \right) \\
&= \hat{D}_i^J \Bigg(1 + \frac{(h_i^J)''}{(h_i^J)'}(p_i - c_i) - \frac{1}{N_f^J} \frac{\{(1-\alpha_I)\alpha_J + \beta_J\beta_I\}\Pi_f^J + \beta_I \Pi_f^I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I} \\
&\quad + \frac{(1-\alpha_I)}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I} \left(\Pi_f^J + \frac{\beta_I}{1-\alpha_I} \frac{N_f^I}{N_f^J} \Pi_f^I \right) \Bigg)
\end{aligned} \tag{102}$$

From this equation, we observe that there exists μ_f^J such that

$$-\frac{(h_i^J)''(p_i)}{(h_i^J)'(p_i)}(p_i - c_i) = \mu_f^J \tag{103}$$

for all $i \in \mathcal{N}_f^J$.

B. Appendix B: Proofs of Propositions

B.1. Proof of Proposition 1

Suppose that T_g is sufficiently close to 0. In both cases with MNL-class and CES-class demand systems, we have $N(0, \alpha) = 0$, the left-hand-side minus the right-hand-side in (15) can be approximated around $\tilde{T}_g = 0$ by

$$\frac{d}{d} N \left(\frac{\gamma(T_f)}{H^*}, \alpha \right) T_g - \frac{d}{d} N (0, \alpha) T_g \tag{104}$$

For each cases with MNL-class and CES-class demand systems, I show that expression 104 is positive.

MNL-class demand In the proof of Proposition 3, I show that in the case with MNL-class demand system, the expression (104) is positive.

CES-class demand From equation FOC-CES, we have

$$\mu_f = \frac{\sigma(1-\alpha)}{\sigma - \alpha - (\sigma-1)N_f}$$

Next, by equation Share-CES, we can write the value of T that leads to the market share N as

$$T = H^{1-\alpha} N^{1-\alpha} \left(1 - \frac{\mu}{\sigma}\right)^{1-\sigma}.$$

Finally, using The Implicit Function Theorem and rearranging, we have

$$\frac{dN}{dT} = \frac{\left(1-\frac{\mu}{\sigma}\right)^{\sigma-1}}{H^{1-\alpha}} N^{\alpha} \left(1 - \frac{\mu_f N_f \frac{\sigma-1}{\sigma(1-\alpha)} \left(1-\frac{\mu_f}{\sigma}\right)^{-1}}{\sigma-\alpha-(\sigma-1)N\left(1-\frac{\mu_f}{\sigma}\right)^{-1}}\right) \to 0 \text{ as } N \to 0.$$

Thus, the condition (15) holds for sufficiently small value of $\tilde{T}_g$.

B.2. Proof of Proposition 2

First, prove the proposition in the MNL case. By the equation $N((\gamma(T)/H^*, \alpha) = 1/|\mathcal{F}|$, we obtain

$$H^* = |\mathcal{F}|\gamma(T)\exp\left(-\frac{|\mathcal{F}|}{|\mathcal{F}|-1}\right) \tag{105}$$

Thus, we have

$$N\left(\frac{\gamma(2T)}{H^*}, \alpha\right) = N\left(2^{\frac{1}{1-\alpha}} \frac{\exp\left(\frac{|\mathcal{F}|}{|\mathcal{F}|-1}\right)}{|\mathcal{F}|}, \alpha\right), \tag{106}$$

which is increasing in α. Further, we have $N(\gamma(2T)/H^*, \alpha) = 2/|\mathcal{F}|$ at $\hat{\alpha}$ such that

$$2^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} = \exp\left(\frac{|\mathcal{F}|}{(|\mathcal{F}|-2)(|\mathcal{F}|-1)}\right). \tag{107}$$

Thus, if $\alpha > \hat{\alpha}$, the merger between two firms improves the consumer surplus. Since $\frac{|\mathcal{F}|}{(|\mathcal{F}|-2)(|\mathcal{F}|-1)}$ decreases with $|\mathcal{F}|$, $\hat{\alpha}$ decreases with $|\mathcal{F}|$.

Next, I show the proposition in the case of CES-class demand. First, the equilibrium level of the aggregator is given by

$$H^* = |\mathcal{F}|\gamma(T)\left(\frac{(\sigma-1)\frac{|\mathcal{F}|-1}{|\mathcal{F}|}}{\sigma-\alpha-\frac{(\sigma-1)}{|\mathcal{F}|}}\right)^{\frac{\sigma-1}{1-\alpha}} \tag{108}$$

Thus, we have

$$N\left(\frac{\gamma(2T)}{H^*},\alpha\right)=N\left(2^{\frac{1}{1-\alpha}}\frac{\left(\frac{(\sigma-1)\frac{|\mathcal{F}|-1}{|\mathcal{F}|}}{\sigma-\alpha-\frac{(\sigma-1)}{|\mathcal{F}|}}\right)^{\frac{1-\sigma}{1-\alpha}}}{|\mathcal{F}|},\alpha\right), \tag{109}$$

Let $\hat{\alpha}$ such that

$$\Lambda(\hat{\alpha},|\mathcal{F}|):=2^{\hat{\alpha}}-\left(\frac{(\sigma-1)\frac{|\mathcal{F}|-1}{|\mathcal{F}|}}{\sigma-\hat{\alpha}-\frac{(\sigma-1)}{|\mathcal{F}|}}\right)^{\sigma-1}=0. \tag{110}$$

Then, a merger between two firms is CS-neutral at $\alpha=\hat{\alpha}$. Since $\Lambda(0,|\mathcal{F}|)<0$ and

$$\frac{\partial\Lambda}{\partial\alpha}=2^{\hat{\alpha}}\log 2+(\sigma-1)2^{-\hat{\alpha}\frac{\sigma}{\sigma-1}}\frac{|\mathcal{F}|-2}{|\mathcal{F}|-1}\frac{\sigma-1}{\mathcal{F}(\sigma-\hat{\alpha})-2(\sigma-1)}>0.$$

at $\alpha=\hat{\alpha}$, the merger between two firms is CS-increases if and only if $\alpha>\hat{\alpha}$. To see that $\hat{\alpha}$ decreases with $|\mathcal{F}|$, I apply the implicit function theorem to obtain

$$\frac{d\hat{\alpha}}{d|\mathcal{F}|}=-\frac{\frac{\partial\Lambda}{\partial|\mathcal{F}|}}{\frac{\partial\Lambda}{\partial\alpha}},$$

where

$$\frac{\partial\Lambda}{\partial|\mathcal{F}|}=(\sigma-1)2^{\alpha\frac{-\sigma}{\sigma-1}}\frac{(1-\hat{\alpha})\left(\sigma-\hat{\alpha}-\frac{2(\sigma-1)}{|\mathcal{F}|^2}\right)}{\left[\left(\sigma-\hat{\alpha}-\frac{2(\sigma-1)}{|\mathcal{F}|}\right)(|\mathcal{F}|-1)\right]^2}>0. \tag{111}$$

B.3. Proof of Proposition 3

First, note that

$$\frac{d}{dT}N_0\left(\frac{\gamma(T)}{H}\right)=\frac{1}{1-\alpha}\frac{1}{T}\frac{\gamma(T)}{H}N_0'\left(\frac{\gamma(T)}{H}\right). \tag{112}$$

Next, suppose that $N_0(\gamma(T)/H)=N$. Then, by the equations FOC-MNL and Share-MNL, we must have

$$N=\frac{\gamma(T)}{H}\exp\left(-\frac{1}{1-N}\right), \tag{113}$$

which implies that

$$T=H^{1-\alpha}N^{1-\alpha}\exp\left(\frac{1-\alpha}{1-N}\right). \tag{114}$$

Thus, we can rewrite $dN_0(\gamma(T)/H)/dT$ as

$$\frac{d}{dT}N_0\left(\frac{\gamma(T)}{H}\right)=\chi(N):=\frac{1}{1-\alpha}H^{\alpha-1}\frac{N^{\alpha}(1-N)^2}{1-N+N^2}\exp\left(-\frac{1-\alpha}{1-N}\right), \tag{115}$$

which approaches to 0 as $N \to 0$ or as $N \to 0$. Further, we have

$$\chi'(N) = \frac{\exp\left(-\frac{1-\alpha}{1-N}\right) N^{\alpha-1}}{(1 - N + N^2)^2} \zeta(N) \tag{116}$$

where

$$\zeta(N) := \left[(1 - N + N^2)\left\{\alpha(1-N)^2 - 2N(1-N) - (1-\alpha)N\right\} + (1-2N)(1-N)^2 N\right]. \tag{117}$$

Note that

$$\zeta(0) = \alpha > 0, \text{ and } \zeta(1) = -(1-\alpha) < 0,$$

which implies that $\xi(N)$ is positive around $N = 0$ and positive around $N = 1$. and

$$\zeta'(0) = -2 - 2\alpha > 0, \text{ and } \zeta'(1) = 2\alpha > 0.$$

By the fact that $\zeta(N)$ is a fourth polynomial of N with negative coefficient of N^4, have only one $\bar{N} \in (0, 1)$ such that $\zeta'(\bar{N}) = 0$. Further, $\zeta'(N) < 0$ for $N \in [0, \bar{N})$ and $\zeta'(N) > 0$ for $N \in (\bar{N}, 1]$. Putting these observations together, we have N^* such that $\zeta(N) \geq$ if and only if $N \leq N^*$. Thus, $\xi(N)$ increases with N if and only if $N \leq N^*$.

The fact that $\xi(0) = \xi(1)$ and that $\xi(N)$ increases with N if and only if $N \leq N^*$ together imply that (i) for any $N, N' \leq N^*$, $\xi(N) > \xi(N')$ if and only if $N > N'$, (ii) for any $N, N' \geq N^*$, $\xi(N) > \xi(N')$ if and only if $N < N'$, and (iii) for any $N < N^*$, there exists $N' > N^*$ such that $\xi(N) = \xi(N')$. These observations leads to the statements of Proposition 3.

B.4. Proof of Proposition 4

B.4.1. Proof of Proposition 4.1 and Proposition 4.2

I first analyze the concavity and the convexity of the function $\tilde{N}'(x)x$ with respect to the network share, and then use this analysis to prove Proposition 4.1 and Proposition 4.2.

In MNL-class demand system, we have

$$N_0'(x)x\Big|_{N_0(x)=N} = \frac{N^3 - 2N^2 + N}{N^2 - N + 1} =: \lambda(N).$$

Then, we have

$$\lambda'(N) = \frac{(1-N)(N^3 - N^2 + 3N - 1)}{(N^2 - N + 1)^2}, \tag{118}$$

which is nonnegative in $N \in [0, \hat{N}]$ and negative in $(\hat{N}, 1]$ for some critical value $\hat{N} \in (0, 1)$ that satisfies

$$\hat{N}^3 - \hat{N}^2 + 3\hat{N} - 1 = 0.$$

Further, Nocke and Schutz (2018a) show that $\lambda(N)$ is concave in N. In summary, $\lambda(N)$ is increasing in $N \in [0, \hat{N})$, nonincreasing in $N \in [\hat{N}, 1]$, and concave in $N \in [0, 1]$

Using this result, I prove Proposition 4.1 and Proposition 4.2.

The required synergies $\hat{\Delta}$ decreases with α if and only if

$$A(T_f + T_g + \hat{\Delta})B(T_f + T_g + \hat{\Delta}) \geq A(T_f)B(T_f) + A(T_g)B(T_g) \tag{119}$$

where

$$A(T_f) = N_0'\left(\frac{\gamma(T)}{H^*}\right)\frac{\gamma(T)}{H^*} \tag{120}$$

and

$$B(T) = \frac{H^*}{\gamma(T)}\frac{d}{d\alpha}\left(\frac{\gamma(T)}{H^*}\right) = \frac{d}{d\alpha}\left(\frac{\gamma(T)}{H^*}\right) = \frac{1}{(1-\alpha)^2}\frac{\sum_{f'\in\mathcal{F}}(\log T - \log T_{f'})N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)}{\sum_{f'\in\mathcal{F}} N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)}. \tag{121}$$

Note that $B(T)$ is increasing in T.

Suppose that two merging firms f and g are weak. If the merged entity is strong, the LHS of (119) is positive while the RHS of (119) is negative. Next, suppose that the merged entity is weak. We have $A(T_f + T_g + \hat{\Delta}) \leq A(T_f) + A(T_g)$ by the concavity of $N_0'(x)x$ in $N \in [0, 1]$. Finally, we have the following inequality

$$\begin{aligned} A(T_f + T_g + \hat{\Delta})B(T_f + T_g + \hat{\Delta}) &\geq (A(T_f) + A(T_g))B(T_f + T_g + \hat{\Delta}) \\ &\geq A(T_f)B(T_f) + A(T_g)B(T_g), \end{aligned} \tag{122}$$

where the last inequality follows from the fact that $B(T)$ is increasing in T and $T_f + T_g + \hat{\Delta} \geq \max\{T_f, T_g\}$. Thus, $\hat{\Delta}$ decreases with α.

Next, I show that $\hat{\Delta}$ for the merger between firm f and g decreases with α if firm f is weak and the firm g is strong and if $N_f + N_g < \hat{N}$. This can be observed by

$$A(T_f + T_g + \hat{\Delta})B(T_f + T_g + \hat{\Delta}) \geq A(T_f)B(T_f) \tag{123}$$

and

$$A(T_g)B(T_g) \leq 0. \tag{124}$$

B.4.2. Proof of Proposition 4.3

When $N_f + N_g \simeq 1$, then $A(T_f + T_g + \hat{\Delta}) \simeq 0$.

$$T_f + T_g + \hat{\Delta} = \left[H^*(N_f + N_g)\exp\left(\frac{1}{1 - N_f - N_g}\right)\right]^{1-\alpha}. \tag{125}$$

Thus, we have

$$\log(T_f + T_g + \hat{\Delta}) = (1-\alpha)\left[\log H^* + \log(N_f + N_g) + \frac{1}{1 - N_f - N_g}\right]$$

As a result,

$$N'(x)x\log(T_f+T_g+\hat{\Delta})\to 0 \text{ as } N_f+N_g\to 1. \tag{126}$$

Thus, $A(T_f+T_g+\hat{\Delta})B(T_f+T_g+\hat{\Delta})\to 0$ as $N_f+N_g\to 1$. Thus, if two merging firms are strong, the LHS of the equation (119) is zero, while the RHS of the equation (119) is positive. Thus, $\hat{\Delta}$ increases with α.

B.5. Proof of Proposition 5

Let $\Pi(\gamma(T)/H)$ be the profit of a firm with is type $\gamma(T)$ at the value of aggregator H. The profit of a firm with the market share N is given by

$$\pi(N)=\Pi\left(\frac{\gamma(T)}{H}\right)\bigg|_{N_0(\gamma(T)/H)=N}=(1-\alpha)\frac{N}{1-N}.$$

Thus, we have

$$\frac{d\Pi}{dT}=\frac{dN}{dT}\frac{d\pi}{dN}=\frac{dN}{dT}\frac{1}{(1-N)^2}. \tag{127}$$

Next, I consider the change in the equilibrium value of the aggregator H^* with respect to the change in the type T_f of firm f. Using The Implicit Function Theorem to the equation 14, we obtain

$$\frac{dH^*}{d\gamma(T_f)}=\frac{N_0'(x_f)}{\sum_g x_g N_0'(x_g)},$$

where $x_g=\gamma(T_f)/H^*$ for each $g\in\mathcal{F}$. Thus, we have

$$\frac{d}{d\gamma(T_f)}\left(\frac{\gamma(T_f)}{H^*}\right)=\frac{1}{H}\left(1-\frac{x_f N_0'(x_f)}{\sum_g x_g N_0'(x_g)}\right).$$

Finally, we obtain

$$\begin{aligned}\frac{d\Pi_f}{dT_f}&=\frac{1}{1-\alpha}\frac{1}{T_f}\frac{\gamma(T_f)}{H}N_0'(x_f)\frac{\sum_{g\neq f}x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)}\\&=\frac{\chi(N_0(x_f))}{(1-N_0(x_f))^2}\frac{\sum_{g\neq f}x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)}=\frac{1}{1-\alpha}H^{\alpha-1}\frac{N^\alpha}{1-N+N^2}\exp\left(-\frac{1-\alpha}{1-N}\right)\frac{\sum_{g\neq f}x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)}.\end{aligned} \tag{128}$$

Drawing on these preliminaries, I show that if T_E is sufficiently small, $\Delta\pi^I>\Delta\Pi^E$ holds. When T_E is sufficiently small, $\Delta\pi^I$ can be approximated as

$$\Delta\pi^I\simeq\frac{d\Pi(\gamma(T_I)/H_0)}{dT_I}T_E>0$$

and $\Delta\pi^E$ can be approximated as

$$\Delta\pi^E\simeq\frac{d\Pi(0)}{dT_E}T_E=0,$$

which implies that $\Delta\pi^I > \Delta\pi^E$ for sufficiently small values of T_E.

Next, I show that when T_E is sufficiently large, $\Delta\pi^I < \Delta\pi^E$ holds. Since $N(\gamma(T)/H) \to 1$ as $T \to \infty$, $H \to \infty$ as $T \to \infty$ and

$$\frac{\sum_{g\neq f} x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)} \to 1 \quad \text{as } T_f \to \infty,$$

we obtain $\lim_{T_f\to\infty} d\Pi_f/dT_f = 0$. As a result, we have $\Pi^E \simeq \Pi^I$ when $T_E \simeq \infty$, and thus

$$\Delta\pi^I = \Delta\pi^E - \Pi_0^I < \Delta\pi^E.$$

B.6. Proof of Lemma 3

To see the effects of T_f^A and T_f^B on n_f^A and n_f^B note that

$$\begin{aligned}
\frac{\partial m^A}{\partial T_f^A} &= \frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}\left[\mu_f^A\left(n_f^B\mu_f^B + 1 - n_f^B + \beta_A\frac{n_f^A}{n_f^B}\right) + \mu_f^A\beta_A^2\frac{n_f^A}{n_f^B}\right] > 0\\
\frac{\partial m^B}{\partial T_f^A} &= -\frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}\left(\beta_A\frac{n_f^A}{n_f^B}\mu_f^A(1-\beta_A) + \beta_A\frac{1-n_f^A}{n_f^B}\right) < 0,
\end{aligned}$$

and thus

$$\frac{\partial N^A}{\partial T_f^A} = \frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}(1-n_f^A)\left(n_f^B\mu_f^B + 1 - n_f^B + \beta_A(1+\beta_A)\frac{n_f^A}{n_f^B}\right) < 0 \tag{129}$$

$$\frac{\partial N^B}{\partial T_f^A} = \frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}\left(\beta_A\frac{n_f^A}{n_f^B}\mu_f^A(1-\beta_A) + \beta_A\frac{1-n_f^A}{n_f^B}\right)n_f^B < 0. \tag{130}$$

Further, note that

$$\begin{aligned}
\frac{\partial m^A}{\partial T_f^B} &= \frac{1}{T_f^B}\frac{1}{\det(G_f)}\left(\beta_A n_f^A\mu_f^A(1-n_f^B) + \beta_A^3\frac{n_f^A}{n_f^B}n_f^A\mu_f^A\right) > 0\\
\frac{\partial m^B}{\partial T_f^B} &= \frac{1}{T_f^B}\frac{1}{\det(G_f)}\left[(n_f^A\mu_f^A + 1 - n_f^A)\left(n_f^B\mu_f^B + \beta_A(1-\beta_A)\frac{n_f^A}{n_f^B}\right) + \beta_A^3 n_f^A\mu_f^A\frac{n_f^A}{n_f^B}\right] > 0,
\end{aligned}$$

and thus

$$\frac{\partial N^A}{\partial T_f^B} = \beta_A\frac{n_f^A}{T_f^B}\frac{1}{\det(G_f)}(1-n_f^A)\left(1 - n_f^B + \beta_A^2\frac{n_f^A}{n_f^B}\right) > 0 \tag{131}$$

$$\frac{\partial N^B}{\partial T_f^B} = \frac{n_f^B}{T_f^B}\frac{1}{\det(G_f)}(n_f^A\mu_f^A + 1 - n_f^A)\left(1 - n_f^B + \beta_A^2\frac{n_f^A}{n_f^B}\right) > 0 \tag{132}$$

B.7. Proof of Proposition 6

First, consider the effect of $x_f \in \{T_f^A, T_f^B\}$ on H^A. By the Implicit Function Theorem, we have

$$\begin{pmatrix} \sum \frac{\partial N_f^A}{\partial H^A} & \sum \frac{\partial N_f^A}{\partial H^B} \\ \sum \frac{\partial N_f^B}{\partial H^A} & \sum \frac{\partial N_f^B}{\partial H^B} \end{pmatrix} \begin{pmatrix} \frac{dH^A}{dx_f} \\ \frac{dH^B}{dx_f} \end{pmatrix} = - \begin{pmatrix} \frac{\partial N_f^A}{\partial x_f} \\ \frac{\partial N_f^B}{\partial x_f} \end{pmatrix}, \tag{133}$$

for $x_f \in \{T_f^A, T_f^B\}$. Using Cramer's rule, we obtain

$$\text{sign}\left(\frac{\partial H^A}{\partial x_f}\right) = \text{sign}\left[-\frac{\partial N_f^A}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^B}{\partial H^B}\right) + \frac{\partial N_f^B}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^A}{\partial H^B}\right)\right] \tag{134}$$

Since $\partial N_f^A / \partial H^B > 0$ for all $f \in \mathcal{F}$ and $\partial N_f^J / \partial x_f > 0$ for any $J \in \{A, B\}$ and $x_f \in \{T_f^A, T_f^B\}$, we have $\partial H^A / \partial T_f^A > 0$ and $\partial H^B / \partial T_f^B > 0$.

Next, consider the effects of T_f^A and T_f^B on H^B. Using the Cramer's rule, we have

$$\text{sign}\left(\frac{\partial H^B}{\partial x_f}\right) = \text{sign}\left[-\frac{\partial N_f^B}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^A}{\partial H^A}\right) + \frac{\partial N_f^A}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^B}{\partial H^A}\right)\right] \tag{135}$$

for $x_f \in \{T_f^A, T_f^B\}$. A calculation shows that $\partial N_f^B / \partial T_f^B > \partial N_f^A / \partial T_f^B$, and that $|\partial N_f^A / \partial H^A| > |\partial N_f^B / \partial H^A|$. These jointly imply that $dH^B / dT_f^B > 0$. Finally, a further calculation shows that

$$\begin{aligned} &\text{sign}\left(\frac{dH^B}{dT_f^A}\right) \\ =&\text{sign}\left[\frac{n_f^A \mu_f^A (1-\beta) + 1 - n_f^A}{(1-n_f^A)\left(n_f^B \mu_f^B + 1 - n_f^B + \beta(1+\beta)\frac{n_f^A}{n_f^B}\right)} - \frac{\sum_{f' \in \mathcal{F}} \left(n_{f'}^A \mu_{f'}^A (1-\beta) + 1 - n_{f'}^A\right)}{\sum_{f' \in \mathcal{F}} \left[(1-n_{f'}^A)\left(n_{f'}^B \mu_{f'}^B + 1 - n_{f'}^B + \beta(1+\beta)\frac{n_{f'}^A}{n_{f'}^B}\right)\right]}\right] \end{aligned} \tag{136}$$

When $n_f^A \simeq 1$, the first item approaches to ∞, while the second item remains to be finite. Thus, if $n_f^A \simeq 1$, $dH^B / dT_f^A > 0$.

Finally, I provide an example where H^B decreases with T_f^A. For simplicity, suppose that $\beta \simeq 0$. Then, we have

$$\begin{aligned} &\text{sign}\left(\frac{dH^B}{dT_f^A}\right) \\ \simeq&\text{sign}\left[\frac{n_f^A \mu_f^A + 1 - n_f^A}{(1-n_f^A)\left(n_f^B \mu_f^B + 1 - n_f^B\right)} - \frac{\sum_{f' \in \mathcal{F}} \left(n_{f'}^A \mu_{f'}^A + 1 - n_{f'}^A\right)}{\sum_{f' \in \mathcal{F}} \left[(1-n_{f'}^A)\left(n_{f'}^B \mu_{f'}^B + 1 - n_{f'}^B\right)\right]}\right] \end{aligned} \tag{137}$$

Consider further the case where all firm but firm f are symmetric, and firm f's shares are given by $n_f^A \simeq 0$ and $n_f^B \simeq 0$. Then, the terms in the brackets in the second line of equation (137) can be rewritten as

$$1 - \frac{1 + \frac{|\mathcal{F}|^2 - |\mathcal{F}| + 1}{|\mathcal{F}| - 1}}{1 + \frac{|\mathcal{F}|^2 - |\mathcal{F}| + 1}{|\mathcal{F}|}} < 0.$$

B.8. Proof of Lemma 4

Let N_k^J be the equilibrium market share of firm $k \in \{f, g\}$ on side $J \in \{A, B\}$. Let $N_M^J = N_f^J + N_g^J$ for each $J \in \{A, B\}$. Consider the merging entity's type (T_M^A, T_M^B) such that

$$\begin{aligned} N^A(T_M^A, T_M^B, H^A, H^B) &= N_M^A \\ N^B(T_M^A, T_M^B, H^A, H^B) &= N_M^B \end{aligned} \tag{138}$$

By the first-order condition, we obtain

$$\mu_M^A = \frac{1}{1 - N_M^A} \tag{139}$$

$$1 - (1 - N_M^B)\mu_M^B - \beta \frac{N_M^A}{N_M^B} = 0. \tag{140}$$

Thus, we obtain the unique markup such that the market shares given the aggregators H^A and H^B is given by the unique pair of μ_M^A and μ_M^B respectively. Finally, the pair (T_M^A, T_M^B) must satisfy the following system of equations:

$$\begin{aligned} N_M^A &= \frac{T_M^A (T_M^B)^\beta \exp(-\mu_M^A - \beta \mu_M^B)}{H^A} \\ N_M^B &= \frac{T_M^B \exp(-\mu_M^B)}{H^B}. \end{aligned} \tag{141}$$

By the latter equation, T_M^B is uniquely determined. Further, once T_M^B is given, T_M^A is also uniquely determined. Let $(\hat{T}_M^A, \hat{T}_M^B)$ be such pair of types. As a result, there is a unique pair of technological synergies $\hat{\Delta}_M^J := \hat{T}_M^J - T_f^J - T_g^J$.

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