The Value of Privacy in Cartels: An Analysis of the Inner Workings of a Bidding Ring

Kei Kawai   Jun Nakabayashi   Juan Ortner[*][†]

U.C. Berkeley   Kindai University   Boston University

July 24, 2021

Abstract

We study the incentives of firms in a bidding cartel that allocates contracts via rotation. We show that a cartel can attain higher payoffs by having the preselected winner randomize its bid and keeping it secret from other members when defection is a concern. Intuitively, randomization makes defection less attractive as potential defectors risk not winning the auction even if they deviate. We illustrate how our theoretical predictions are borne out in practice by studying a bidding cartel that operated in Kumatori, Japan. Our case study offers a concrete instance in which randomization is used to relax incentive compatibility constraints.

KEYWORDS: procurement, collusion, bidding ring, cartel, privacy.

1 Introduction

Consider the problem of a cartel member who has been pre-selected as the winner of an upcoming first-price sealed-bid procurement auction. The objective of the pre-selected winner is to be the lowest bidder and, at the same time, submit an inflated bid to secure high margins. The extent to which the bid can be inflated is limited by both the reserve price and the incentive compatibility (IC) constraints of the losing cartel bidders who may defect. This paper studies both theoretically and empirically the problem of how the cartel should bid in this scenario. We show that, when the IC constraints are binding, the pre-selected winner should randomize its bid and keep it secret from the designated losers. We then show that our theoretical predictions are borne out in practice through a case study of a detected bidding cartel in Japan. Our case study offers a concrete example in which randomization is used to relax incentive compatibility constraints.

The analysis of the paper is based on the simple intuition that, by keeping its bid secret, the pre-selected winner can make it harder for other bidders to defect. If the pre-selected winner's bid is publicly known, a defector can undercut the announced bid to obtain maximal deviation profits. If instead the pre-selected winner's bid is unknown, a potential defector runs the risk of deviating (and be punished) without even winning the auction.

We formalize these ideas with a model in which a group of firms repeatedly plays a first-price procurement auction. The size of the project auctioned off is drawn i.i.d. each period, and firms share the same costs. Consistent with our empirical application, we assume that the cartel has access to a mediator and allocates contracts through a bid rotation scheme. We say that an equilibrium has *common-knowledge bids* if firms' bids depend deterministically on the public history.

Under the cartel's optimal bid rotation equilibrium with common-knowledge bids, winning bids are determined by either the losers' IC constraints or the reserve price. As in Rotemberg and Saloner (1986), winning bids are below the reserve price whenever the project is sufficiently large.

Our main result shows that a cartel may strictly gain from bidding schemes without

common-knowledge bids. Under the optimal bid rotation equilibrium, the mediator randomly draws the winner's bid from a distribution F, and privately communicates this bid to the pre-selected winner. Non-winners are instructed to place a losing bid. Distribution F has the property that, under this equilibrium, non-winners are indifferent between placing a losing bid or deviating and placing any bid in the support of F. By keeping losing firms uninformed, such a bidding scheme relaxes incentive constraints and yields larger profits for the cartel.

Our theory delivers two key predictions. First, since the incentive constraints of losing firms are more likely to bind for large projects, collusive schemes with randomized bids are most profitable (and hence most likely used) when the project auctioned off is large. Second, optimal randomized bidding schemes tend to have a sizable gap between the winning bid and the second lowest bid; i.e., there is significant amount of money left on the table.

In order to illustrate how these theoretical forces are borne out in practice, we study the inner workings of a bidding ring consisting of mid and small contractors participating in procurement auctions let by the town of Kumatori, Japan. The cartel was active until October 2007, when members of the cartel were investigated and prosecuted for bid-rigging. The cartel case in Kumatori makes it ideal for studying the incentives of the members of the bidding ring because the criminal case and the subsequent liability claims case both went to trial. The court proceedings produced a wealth of information. The rulings of the case alone offer detailed and rich descriptions about the actions of the cartel members as well as the motivations behind their actions.

Another useful aspect of the collusion case is that there was significant variation in the size of projects let by the town. In particular, there was one occasion in which cartel members bid on a project (building a housing complex) that was about 17 times the size of the average project let by the town. The variation in project size induces variation in the IC constraints of losing cartel bidders as temptation to defect is typically bigger for larger projects. The variation in the IC constraint is useful for validating the theory. Consistent with the predictions of our model, we find that the pre-selected winner took extra precautions

to keep its bid secret when bidding on the housing complex. Court documents show that the designated losers were instructed how to bid but were kept in the dark as to how the designated winner would bid. Moreover, the pre-selected bidder bid much lower than what the losing bidders were instructed to bid. The winner's bid was about 11.5% lower than the next lowest bid, while the average difference between the winning bid and the losing bids for other auctions was less than 1%. The court ruling describes the motive of the pre-selected winner for leaving money on the table in that auction as trying to make it difficult for other cartel members to guess its bid.

Our analysis highlights the practical value of randomization and privacy in relaxing IC constraints in naturally occurring settings. While the idea that randomization can help relax IC constraints appears broadly in theoretical work on mechanism design and repeated games, empirical evidence on the use of such randomization has largely been limited to audits, such as tax audits and police traffic stops. Our case study offers a concrete instance in which randomization is used to relax IC constraints outside of audits. Our analysis suggests that this idea is broadly useful in positive theories of behavior.

More specific to the antitrust context, our analysis sheds light on the role that transparency plays in sustaining a successful collusive scheme. It is well known from the theory of repeated games that transparency allows colluders to better coordinate and monitor each others' actions. However, motivated by the way communication was structured in several recently detected cartels, Sugaya and Wolitzky (2018) identify a potential drawback of transparency: they show that transparency may hinder collusion by enabling potential defectors to devise more profitable deviations. Our case study is a concrete example of how privacy helps cartels. By keeping its bid secret from other bidders, the pre-selected winner makes it harder for potential defectors to profitably deviate.

The paper has implications for screens of collusion, in particular, the price-variance screen proposed by Abrantes-Metz et al. (2006). The variance screen flags unusually low variance of the price sequence (or equivalently, high degrees of price stability) as a marker for collusion.[1]

---

[1]Prior work that makes the connection between competition and stability of prices include Carlton (1986) and Levenstein (1997).

The price-variance screen was originally motivated by the observation that variance in prices increased substantially after the collapse of a bidding cartel consisting of seafood processors. Researchers have since found similar patterns in other settings such as Swiss procurement auctions (Imhof et al., 2016) and LIBOR (Abrantes-Metz et al., 2012), as well as in non-auction markets.[2] Our study provides a better understanding of when low price variance is likely to be associated with collusive bidding (when IC constraints don't bind, and all bids are clustered around the reserve price) and when it might not (when IC constraints bind).

Lastly, the paper also has implications for the information that an auctioneer facing collusive bidders should make public after each auction. In particular, while randomized bids can be profitable for a cartel when all bids are public, such strategies are no longer beneficial when only the winning bid is public. As a result, only revealing the winning bid can alleviate collusion and lead to strictly lower prices. Intuitively, if only the winning bid is public, deviations are only detected when the defector wins the auction. This contrasts with settings in which all bids are public, in which defectors can be detected and punished even when they don't win.

Related literature. Previous papers have highlighted how randomization and privacy may help relax incentive constraints. Rahman and Obara (2010) and Rahman (2012) show that randomized messages can help provide incentives by making it easier to identify deviators. Jehiel (2015) establishes conditions under which a principal may benefit from keeping her agent in the dark about payoff relevant parameters. Ederer et al. (2018) show that randomized contracts may help prevent gaming by agents. Ortner and Chassang (2018) and Chassang and Miquel (2019) illustrate how randomized incentive schemes may help deter collusion by introducing contracting frictions among collusive parties. Within repeated-game settings, Kandori (2003), Kandori and Obara (2006), and Rahman (2014) show how randomization may allow arbitrarily patient players to sustain larger equilibrium payoffs.[3]

More specifically, our paper relates to previous work on how cartels bid in a repeated-

---

[2]Examples include retail gasoline and pasta products (Jiménez and Perdiguero, 2012, Crede, 2019)

[3]Relatedly, Bernheim and Madsen (2017) show that a cartel's optimal stationary collusive scheme may involve randomization.

game setting. There is a large theoretical literature on this topic exploring issues such as monitoring (e.g., Green and Porter, 1984, Skrzypacz and Hopenhayn, 2004, Sugaya and Wolitzky, 2018), efficiency and private costs (e.g., Athey and Bagwell, 2001, Athey et al., 2004, Athey and Bagwell, 2008) and demand shocks (e.g., Rotemberg and Saloner, 1986, Haltiwanger and Harrington, 1991). Our work also relates to prior papers studying how communication may help sustain collusion (e.g., Compte, 1998, Kandori and Matsushima, 1998, Harrington and Skrzypacz, 2011, Rahman, 2014, Awaya and Krishna, 2016).

The empirical portion of our work is closely related to studies that test whether or not the price patterns implied by models of collusion are borne out in the data. Porter (1983) and Ellison (1994) study pricing by the Joint Executive Committee to test for the models of Green and Porter (1984) and Rotemberg and Saloner (1986). Levenstein (1997) studies the bromine cartel and finds evidence consistent with the model of Green and Porter (1984). Borenstein and Shepard (1996) study the retail gasoline market to test for the model of Haltiwanger and Harrington (1991). Wang (2009) also studies the retail gasoline market and finds evidence consistent with the equilibrium of Maskin and Tirole (1988). Wang (2009) is also one of the first papers to document evidence of mixed strategy equilibria for non zero-sum games outside of the lab. Earlier work that documents the use of mixed strategies for zero-sum games include Walker and Wooders (2001) and Chiappori et al. (2002).

Our paper is also related to the literature on detecting instances of collusion. Early seminal work includes Hendricks and Porter (1988), Baldwin et al. (1997) and Porter and Zona (1993, 1999). More recent work includes Bajari and Ye (2003), Abrantes-Metz et al. (2006), Athey et al. (2011), Conley and Decarolis (2016), Schurter (2017), Kawai and Nakabayashi (2018), and Chassang et al. (2020).[4]

Lastly, our results indirectly relate to the literature on information design (Kamenica and Gentzkow, 2011). In particular, the indifference condition characterizing the optimal

---

[4]Other related work includes Pesendorfer (2000), who studies bidding rings with and without side-payments, and Asker (2010), who studies knockout auctions among cartel members. Ohashi (2009) and Chassang and Ortner (2019) document how changes in auction design can affect the ability of bidders to sustain collusion. Clark et al. (2018) analyze the breakdown of a cartel and its implications on prices. For surveys of the literature, see Porter (2005) and Harrington (2008).

bid distribution in our model has analogs in Roesler and Szentes (2017) and Condorelli and Szentes (2020), who study information acquisition by a buyer, Perez-Richet and Skreta (2018), who study test design, and Ortner and Chassang (2018), who study the design of anti-corruption schemes.

2 Bid-rigging in the Town of Kumatori

This section provides a brief description of the bidding ring that operated in the town of Kumatori until October 2007.

2.1 Background

Auctions for construction projects in Kumatori. The town of Kumatori uses auctions to allocate construction projects that are estimated to cost more than 1.3 million yen, or about 13 thousand dollars. The auction format is first-price sealed bid with a secret reserve price, although the reserve price was not binding in any of the auctions that we study.

An important feature of the auctions is that participation is by invitation only. The town maintains a list of qualified contractors and invites a subset of firms from the list to bid. The town maintains separate lists for projects with different sizes. For example, for building construction, the town maintains four separate lists of contractors (Tier A through Tier D). The town typically invites Tier A firms to bid on the largest projects, Tier B firms to bid on the next largest projects, and so on, essentially segmenting the market by project size. All of Tier A firms are headquartered outside of Kumatori and are invited to bid only on exceptionally large projects. Tier B firms and below are local firms typically headquartered within the town. Most of the Tier B and C firms were members of the Kumatori Contractors Cooperative, a trade association that consisted of a little more than 20 mid and small size contractors in Kumatori. The members of this cooperative were found to be colluding.

Bidding ring in the town of Kumatori. Reports of police investigation of the members of the Kumatori Contractors Cooperative for bid-rigging first appeared in the news on

7

October 12, 2007. In addition to the contractors in Kumatori, more than 20 town officials, including the town mayor, were questioned by the police. In November 2007, four individuals were indicted for bid-rigging. The criminal charges focused on the defendants' involvement in bid-rigging in a single auction that took place on August 22, 2006 that Imakatsu Construction won, an auction for rebuilding a public housing complex (Ohara Residences). The defendants included Mr. Kitagawa, the owner of Imakatsu Construction and director of the Kumatori Contractors Cooperative; his son, who was an employee of Imakatsu Construction; Mr. Nishio, the vice-director of the Cooperative; and Mr. Takano, an employee of the Cooperative. While Mr. Nishio and Mr. Takano were not participants of the auction, they mediated much of the communication between Mr. Kitagawa and the other participants of the auction. For example, Mr. Nishio gave out the instructions to other bidders on how they should bid. All four defendants were found guilty in trial in March 2008. Mr. Kitagawa was sentenced to prison for 18 months, Mr. Nishio to 14 months, and the other defendants were sentenced to 10 months.

Although the criminal case focused on the defendants' involvement in bid-rigging in the Ohara Residences auction, the court ruling indicated that the Ohara Residences auction was not an isolated case and that the participants of the bid-rigging scheme were not confined to those that were criminally charged.[5] In the ruling, the court stated that the members of the Kumatori Contractors Cooperative allocated the projects according to a predetermined order to even out the work of each contractor. While none of the town officials were formally charged, the court ruling stated that the designated winner of the cartel would approach the town officials to seek out the engineering estimate.

In response to the ruling of the criminal case, the town of Kumatori withheld part of its payment to Imakatsu Construction for work that had been completed on the Ohara residences in order to off-set liability claims. However, the mayor and town officials showed little interest in pursuing claims for damages incurred on other auctions. This inaction led some of the residents of Kumatori to file suit against the mayor asking the court to order

---

[5]Our description of the cartel in this paragraph is taken from page 2 and 3 of Ruling H19 (WA) No. 6418, Osaka District Court.

the mayor to pursue claims against 23 firms, all members of the Kumatori Contractors Cooperative, for damages incurred on other auctions. The District Court of Osaka ruled in favor of the plaintiffs, ordering the town to pursue claims against the bidders in the amount of about 375 million yen, or about 3.75 million dollars. The mayor appealed the ruling, but the verdict was upheld by the Osaka High Court with relatively minor modifications.

2.2   Ohara Residences Auction

Among the auctions that the cartel bid on, the Ohara residences auction was unique because of the large value of the project. This section discusses briefly how, despite the town's policy of segmenting the market by project size, the members of the Kumatori Contractors Cooperative were invited to bid in the auction.

The town of Kumatori started planning for the rebuilding of the Ohara Residence Complex, an ageing public housing project, around year 2000 according to the minutes of the meeting of the town council.[6] The new residence complex would consist of three separate buildings. Construction of the first and smallest of the three buildings was put to an auction in April 2004 to Tier A firms. The winning bid was 363 million yen, or about 3.6 million dollars. The winner of the auction was Asanuma Corporation, a contractor headquartered in the city of Osaka with annual sales of about 2 billion dollars in FY 2005.

In the Fall of 2004, Mr. Kitagawa, the director of the Kumatori Contractors Cooperative and owner of Imakatsu Construction began lobbying the town's mayor and the head of the town's general affairs department to let Tier B firms bid on the second and third components of the Ohara Residences Complex.[7] Mr. Kitagawa was an important supporter of the mayor. Despite the repeated lobbying by Mr. Kitagawa, the head of the department was reluctant to let Tier B firms bid on the Ohara residences project initially, according to court documents. However, the head of the department started to warm to the idea around April 2006.[8] On August 3, 2006, the town office invited five contractors to bid on the housing complex project,

---

[6]Minutes of the meeting of the town countil to discuss the budget, March 2001, page 49.

[7]Our description of the cartel in this paragraph are taken from page 17 of Ruling H21 (Gyo-U) No. 99, Osaka District Court.

[8]Last paragraph, page 17 of the Osaka District Court ruling.

including Imakatsu Construction. All of the five invited bidders were Tier B firms and also members of the Cooperative.

3 Motivating Evidence

In this section, we briefly discuss the data and document bidding patterns of the cartel that motivate our theoretical and qualitative analysis in sections 4 through 6.

3.1 Data

Our primary data source is the bid data submitted to court by the residents of Kumatori who sued the mayor for failing to pursue damages against the bidding ring. The residents claimed that the town was owed money for a subset of the auctions won by the bidding ring from April 2003 to October 2007. We have information on all of the bids, date of the auction, reserve price, and the identity of the winner (but not the identity of the losing bidders) for those auctions. A limitation of this dataset is that it does not include auctions for which the plaintiffs did not pursue damages even if a cartel member won the auction. Moreover, lettings that were awarded to non-colluding firms are not included.

We complement our primary data source with auction data that were collected by the town of Kumatori. This dataset covers all auctions let by the town between April 2006 and December 2009. Because the second dataset includes the universe of auctions let by the town during this period, the second dataset is a superset of the first during the period the two datasets overlap, i.e., April 2006 and October 2007. The second dataset contains information on all bids, identity of the bidders, date of the auction and the reserve price.

Table 1 reports the sample statistics of the two datasets. The top panel reports the summary statistics without conditioning on the tier of the participants and the bottom panel focuses on auctions in which Tier B firms were invited to bid. As we discussed in section 2.1, the town of Kumatori segments the market by the size of the project and invites

	Data from Plaintiffs		Data from Kumatori
	All periods	2006.4 - 2007.10	2006.4 - 2007.10
	(1)	(2)	(3)
All Tiers			
Reserve price (mil. Yen)	28.93 (58.48)	39.65 (105.07)	31.33 (79.97)
Lowest bid (mil. Yen)	28.06 (56.44)	38.27 (100.65)	29.21 (75.67)
Lowest bid/Reserve	0.967 (0.011)	0.964 (0.011)	0.928 (0.068)
#Bidders	10.41 (2.76)	8.00 (2.77)	9.41 (2.68)
Sample size	158	39	79
Tier B Auctions			
Reserve price (mil. Yen)	47.80 (87.97)	166.83 (244.92)	158.63 (224.63)
Lowest bid (mil. Yen)	46.46 (84.79)	161.17 (234.60)	152.69 (215.33)
Lowest bid/Reserve	0.969 (0.011)	0.971 (0.015)	0.965 (0.020)
#Bidders	10.84 (2.78)	8.33 (1.63)	8.57 (1.62)
Sample size	63	6	7

Table 1: Sample Statistics – Auctions. Columns (1) and (2) correspond to the sample of auctions in the plaintiffs dataset and column (3) corresponds to those in the dataset we obtained from the town. Column (2) corresponds to a subset of the auctions in the plaintiffs dataset that were let between April 2006 and October 2007. The bottom panel corresponds to Tier B auctions. Standard errors are reported in parenthesis.

bidders from a particular tier to bid on a given auction. We are specifically interested in Tier B firms because the participants of the Ohara residence auction were Tier B firms and these are the firms we know the most about from the court documents. All of the Tier B firms were members of the bidding ring.

The first two columns of Table 1 correspond to the dataset we obtained from the plaintiffs. Column (1) reports the summary statistics for all of the auctions in that sample (top panel) and those for Tier B auctions (bottom panel). Column (2) reports the summary statistics for the subset of the auctions that were let after April 2006, which is when the dataset we obtained from the town begins. We find that the average reserve price is 28.93 million yen (about $290,000) in the top panel of column (1) while it is significantly higher, at 39.65 million yen in the top panel of column (2). The difference in the mean reserve price between column (1) and column (2) is explained by the fact that the Ohara residence auction is in the sample that corresponds to column (2). The reserve price of the Ohara residences auction was 657 million yen which is about 17 times larger than the other auctions. This auction raises the average reserve price and the winning bid much more in column (2) given the smaller sample size in column (2). We also find that the reserve price and the bids in the bottom panel are larger than in the top panel, reflecting the fact that Tier B auctions are generally larger than Tier C auctions. Note that the data from the plaintiffs only include Tier B and Tier C auctions. The average winning bid is about 96-7% of the reserve price.

Column (3) reports the sample statistics for the dataset that we obtained from the town of Kumatori. There are 79 auctions that were let by the town between the beginning of the sample (April 2006) and the breakdown of the cartel in October 2007. Because the set of auctions in the town data is not a selected sample, the data include auctions in which the cartel bidders did not participate as well as those in which the cartel bidders participated but are not included in the plaintiffs dataset. Comparing the sample sizes between columns (2) and (3) in the top panel, we find that about half (39/79) of the auctions are included in

Table 2: Tier B Firms

Name	(1) Revenue	(2) #Won	(3) Ohara
Imakatsu Construction	759.45	6	●
Nakabayashi Construction	488.65	12	
Nishio Gumi	454.40	8	
Tokushin construction	416.39	6	
Takada Gumi	396.25	9	○
Yamamoto Construction	195.60	7	○
Nakajima Kougyou	159.80	5	
Hannan Construction	109.70	4	○
Seiko Construction	48.23	7	○

Table 3: Sample Statistics – Tier B firms.

the first dataset during the period of overlap. If we focus on the bottom panel, we find that the sample size is 6 in column (2) and 7 in column (3). The fact that 6 out of 7 auctions let during the period of overlap are in the first dataset suggests that that sample selection issues are unlikely to be too severe for Tier B auctions.

Table 3 reports the summary statistics of the 9 contractors that were on Tier B.[9] Column (1) reports the total amount awarded to each of the firms during the sample period. The award amount varies from a high of about 760 million yen to a low of about 50 million yen. The number of auctions awarded is reported in column (2). Column (3) reports whether or not a firm was invited to the Ohara residences auction. A white circle indicates that a firm participated in the auction and a black circle indicates that a firm won the auction.

---

[9]The sample of auctions used for Table 3 include all of the 63 Tier B auctions from the first dataset and one Tier B auction from the town data.

3.2 Size of Auctions and Winning Margin

We now illustrate how distinct the Ohara residences auction was from the other auctions for Tier B firms. The left Panel of Figure 1 plots the reserve price of Tier B auctions. The horizontal axis is the calendar date and the vertical axis is the reserve price of the auction. The vertical dotted line corresponds to the start of the town data. The figure shows that, except for the Ohara residence auction that took place on August of 2006 (corresponding to the dark circle), Tier B firms were invited to bid on auctions with reserve prices below 200 million yen. The average reserve price during this period excluding the Ohara residences auction is about 39.1 million yen. The Ohara residences auction was about 17 times the average size of projects on which these bidders were invited to bid.

The right panel of Figure 1 plots the difference between the lowest and the second lowest bids as a fraction of the reserve price for these auctions. The figure shows that the difference is always less than 4%, except for the Ohara residences auction (corresponding to the dark circle). The average difference for lettings excluding Ohara Residences is about 0.93%. The bid difference for Ohara Residences, on the other hand, is 11.4%. The bidding patterns suggest that the cartel members kept the winning margin small for all projects except for the Ohara residences project. The next sections explore, first theoretically and then qualitatively – through close examination of court documents – how these bidding patterns can be explained as an optimal cartel response when IC constraints of designated losers bind.

4   Model

We now present a simple model of repeated procurement auctions in which bidders collude by bid rotation. The key factors in the model that constrain the winning bid of the cartel are the reserve price and the losers' IC constraints. We show that the equilibrium payoffs
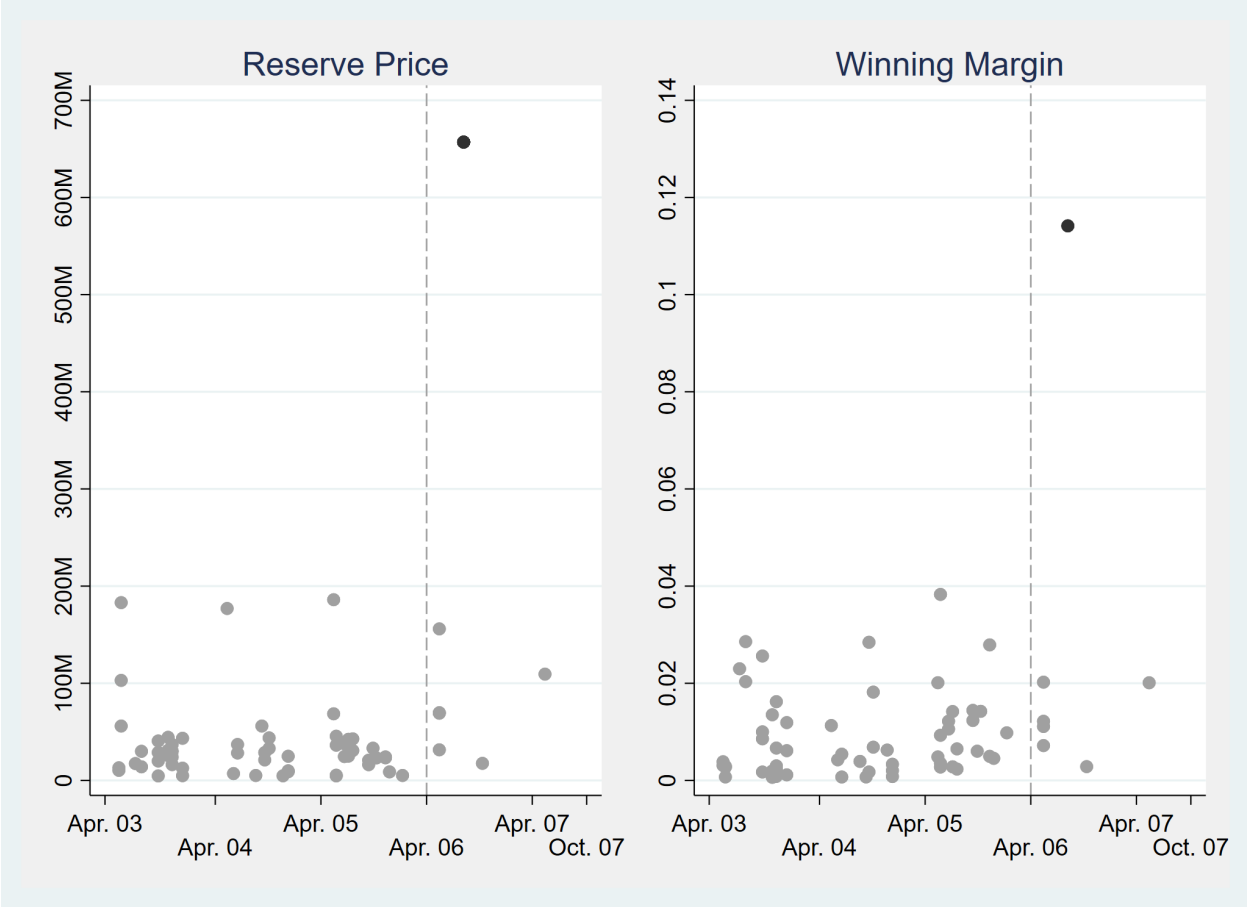
Figure 1: Reserve Price of Auctions (Left Panel) and Difference between Winning Bid and Second Lowest Bid (Right Panel). Left panel of the Figure plots the reserve price of auctions in which collusive firms in Tier B were invited to bid. The right panel plots the difference between the winning bid and the lowest losing bid, as a fraction of reserve prices.

15

of the cartel can be made higher by using strategies that are not deterministic functions of the history when the reserve price is sufficiently large. Under the optimal cartel equilibrium, the wining margin is close to zero when the reserve price is low, but the winning margin can be substantial when the reserve price exceeds some threshold, consistent with the bidding patterns that we document in Figure 1.

Consider a repeated game in which, in each period $t \in \mathbb{N}$, a buyer procures a project from firms $i = 1, 2$. To simplify the exposition, we assume that the buyer uses a first-price auction with a public reserve price. Our results generalize to the case of $n > 2$ bidders, and to auctions with secret reserve prices.

In each period t, firms $i = 1, 2$ share the same procurement cost $c \geq 0$, which we normalize to $c = 0$. Public reserve price r_t is drawn i.i.d. over time from distribution F_r with support $[\underline{r}, \overline{r}]$, with $\overline{r} > \underline{r} \geq 0$. After observing r_t, firms submit public bids $\mathbf{b}_t = (b_{i,t})_{i=1,2}$. This yields allocation $\mathbf{x}_t = (x_{i,t})_{i=1,2} \in [0, 1]^2$ such that: if $b_{j,t} > b_{i,t}$ then $x_{i,t} = 1$; if $b_{j,t} < b_{i,t}$ then $x_{i,t} = 0$. In the case of ties, we follow Athey and Bagwell (2001) and Chassang and Ortner (2019) and let bidders jointly determine the allocation. Formally, we allow bidders $i = 1, 2$ to simultaneously and publicly pick numbers $\gamma_{i,t} \in [0, 1]$. When bids are tied, the allocation to bidder i is $x_{i,t} = \frac{\gamma_{i,t}}{\gamma_{i,t} + \gamma_{j,t}}$.[10] Firm i's profits in period t are $x_{i,t}(b_{i,t} - c) = x_{i,t}b_{i,t}$. Firms share a common discount factor $\delta < 1$.

Mediation. Consistent with our application, we assume that firms have access to a mediator. In each period t, the mediator observes the history of past reserve prices and bids, as well as current reserve price r_t. Prior to bidding, the mediator privately sends recommended bids $(\widehat{b}_{i,t}, \widehat{\gamma}_{i,t})$ to firms $i = 1, 2$. Recommendations $(\widehat{b}_{i,t}, \widehat{\gamma}_{i,t})_{i=1,2}$ may depend on the history of past reserve prices and bids, current reserve price r_t, and the history of past recommendations.[11]

---

[10] If $\gamma_{i,t} = \gamma_{j,t} = 0$, then $x_{i,t} = x_{j,t} = 1/2$.

[11] See Appendix A for a formal description of the mediator's strategies. See also Sugaya and Wolitzky (2017) for a detailed exposition of repeated games with a mediator.

Solution concepts. A period-t history for bidder i,

$$h_{i,t} = (r_s, \widehat{b}_{i,s}, \widehat{\gamma}_{i,s}, \mathbf{b}_s, \boldsymbol{\gamma}_s)_{s<t} \sqcup (r_t, \widehat{b}_{i,t}, \widehat{\gamma}_{i,t}),$$

records past reserve prices $(r_s)_{s<t}$, mediator's recommendations $(\widehat{b}_{i,s}, \widehat{\gamma}_{i,s})_{s<t}$, and realized bids $(\mathbf{b}_s, \boldsymbol{\gamma}_s)_{s<t}$, as well as current reserve price r_t and mediator's recommendation $(\widehat{b}_{i,t}, \widehat{\gamma}_{i,t})$. A pure strategy $\sigma_i : h_{i,t} \mapsto (b_{i,t}, \gamma_{i,t})$ for bidder i maps bidder i histories to bids. Our solution concept is weak Perfect Bayesian Equilibrium, which we simply refer to as equilibrium. The period-t public history is $h_t^0 = (r_s, \mathbf{b}_s, \boldsymbol{\gamma}_s)_{s<t}$.

Definition 1. *We say that equilibrium $\sigma = (\sigma_i)_{i=1,2}$ is a* bid rotation equilibrium *if there exists $i, j = 1, 2$, $i \neq j$, such that for all on-path public histories h_t^0, $\mathbb{E}_\sigma[x_{i,t}|h_t^0] = 1$ if t is even and $\mathbb{E}_\sigma[x_{j,t}|h_t^0] = 1$ if t is odd.*[12]

We focus on bid rotation equilibria, which corresponds to the bidding scheme used by the bidding ring in Kumatori.[13]

Definition 2. *We say that equilibrium $\sigma = (\sigma_i)_{i=1,2}$ has* common-knowledge bids *if, for $i = 1, 2$ and for all histories $h_{i,t}$, $\sigma_i(h_{i,t})$ is a pure action and depends only on h_t^0 and r_t.*

We note that equilibria with common-knowledge bids correspond to pure strategy equilibria of the game without the mediator. Let Σ denote the set of bid rotation equilibria, and let $\Sigma^{\mathsf{CK}} \subset \Sigma$ denote the set of bid rotation equilibria with common-knowledge bids. For each equilibrium σ and $i = 1, 2$, let $V_i(\sigma)$ denote firm i's expected discounted payoff at the start of the game under σ.

---

[12]Formally, an equilibrium in this environment is given by a strategy profile $(\sigma_M, \sigma_1, \sigma_2)$, where σ_M is the mediator's strategy, and bidders' beliefs $\mu = (\mu_1, \mu_2)$ about the messages sent by the mediator; see Appendix A for details. To economize on notation, in the main text we denote an equilibrium simply by the bidders' strategies $\sigma = (\sigma_i)_{i=1,2}$.

[13]We note that bid rotation equilibria would be optimal under a natural generalization of our model, in which firms have (publicly observed) costs that are increasing in their backlog. Indeed, suppose that the procurement cost of last period's winner is $\bar{c} > 0$, and the cost of last period's loser is $\underline{c} \in [0, \bar{c})$. If $\bar{c} - \underline{c}$ is large enough, then the cartel's optimal equilibrium involves bid rotation.

Define

$$\overline{V}^{\mathsf{CK}} \equiv \sup_{\sigma \in \Sigma^{\mathsf{CK}}} \sum_{i=1,2} V_i(\sigma), \text{ and}$$

$$\overline{V} \equiv \sup_{\sigma \in \Sigma} \sum_{i=1,2} V_i(\sigma),$$

to be, respectively, the cartel's largest payoffs under an equilibrium in Σ^{CK} and Σ. Since $\Sigma^{\mathsf{CK}} \subset \Sigma$, we have $\overline{V} \geq \overline{V}^{\mathsf{CK}}$.

Equilibria with common-knowledge bids. Our first result characterizes optimal equilibria in Σ^{CK}. These equilibria take the following intuitive form. The designated bidder bids the reserve price, r_t, or the expected continuation payoff of the designated loser, $\delta \overline{W}^{\mathsf{CK}}$, whichever is less. The designated loser bids a marginally losing bid.[14] Deviations are punished by Nash reversion. Because the designated loser becomes the designated winner next period under bid rotation, $\overline{W}^{\mathsf{CK}}$ is the expected continuation value of the designate winner and we have $\overline{V}^{\mathsf{CK}} = \overline{W}^{\mathsf{CK}} + \delta \overline{W}^{\mathsf{CK}}$. As we show in Appendix B, continuation value $\overline{W}^{\mathsf{CK}}$ is the largest solution $W \geq 0$ to the following equation:

$$W = \frac{1}{1 - \delta^2} \mathbb{E}_{F_r}[\min\{r, \delta W\}]. \tag{1}$$

Proposition 1 formalizes this discussion. All proofs are collected in Appendix B.

Proposition 1. *On the equilibrium path, the bidding strategy in any $\sigma \in \Sigma^{\mathsf{CK}}$ that attains $\overline{V}^{CK}$ is such that, in all periods t, the winning bid is given by the minimum between r_t and $\delta \overline{W}^{CK}$, where $\delta \overline{W}^{CK}$ is the expected continuation payoff of a designated loser under any equilibrium that attains $\overline{V}^{CK}$.*

---

[14]More precisely, the designated loser bids the same bid b as the designated winner but chooses $\gamma = 0$.

Equilibria without common-knowledge bids. Our main result characterizes optimal equilibria in Σ, and establishes when $\overline{V} > \overline{V}^{\mathsf{CK}}$. We describe the optimal equilibrium in words before stating the results in the form of a proposition.

Let $\overline{W}$ be the value function of the designated winner under an equilibrium that attains $\overline{V}$. Because the designated loser becomes the designated winner next period, the expected continuation value of the losing bidder is $\delta\overline{W}$.[15] In any optimal equilibrium in Σ, along the path of play the mediator recommends bid $(b, \gamma = 1)$ to the winner, with b drawn from c.d.f. $F^*(\cdot; r_t)$. The distribution $F^*(\cdot; r_t)$ is degenerate at r_t if $r_t \leq \delta\overline{W}$ and is non-degenerate otherwise. The loser is recommended to bid $(\overline{b}, \gamma = 0)$, where $\overline{b}$ is the largest point in the support of $F^*(\cdot; r_t)$. If either bidder deviates, the mediator sends bidding recommendations $(b_i, \gamma_i) = (0, 1)$ to $i = 1, 2$ from the next period onwards, and players adhere to this recommendation; i.e., they play Bertrand-Nash. Note that, while deviations by the loser are publicly observed, deviations by the winner may only be detected by the mediator (since bidding recommendations are private). The mediator's messages following a deviation provide the winner with incentives to follow the recommended bid.

Next, we show how we derive the distribution $F^*(\cdot; r)$. Recall that the loser's discounted continuation payoff is $\delta\overline{W}$. Suppose that the winning bid at time t is drawn from c.d.f. F_t. Let $\overline{b}$ and $\underline{b}$ denote, respectively, the largest and smallest points in the support of F_t. For the loser not to have an incentive to deviate and place a bid $b < \overline{b}$, F_t must satisfy:

$$\forall b < \overline{b}, \quad (1 - F_t(b))b \leq \delta\overline{W} \iff F_t(b) \geq 1 - \frac{\delta\overline{W}}{b}. \tag{2}$$

Equation (2) implies $\underline{b} \leq \delta\overline{W}$.

Consider now the incentives of the predetermined winner. If the mediator recommends bid $b < \overline{b}$, the winner can increase its bid to $\overline{b} - \epsilon \approx \overline{b}$ and still win the auction. For the

---

[15]We show in Appendix B that optimal equilibria in Σ are stationary.

winner to have incentives to follow the mediator's recommendation, we must have

$$\underline{b} + \delta^2 \overline{W} \geq \overline{b}, \tag{3}$$

where the inequality follows since the winner's equilibrium continuation payoff is $\delta^2 \overline{W}$. Since $\underline{b} \leq \delta \overline{W}$ (by equation (2)), inequality (3) gives us $\overline{b} \leq \delta(1+\delta)\overline{W}$. Distribution $F^*(\cdot; r)$ is the highest distribution (in terms of f.o.s.d.) with $\overline{b} \leq r$ satisfying (2) and (3). When $\delta \overline{W} \geq r$, $F^*(\cdot; r)$ puts all its mass at r. When $\delta \overline{W} < r$, $F^*(\cdot; r)$ is given by:

$$F^*(b; r) = \begin{cases} 0 \text{ if } b < \delta \overline{W}, \\ 1 - \frac{\delta \overline{W}}{b} \text{ if } b \in [\delta \overline{W}, \min\{r, \delta(1+\delta)\overline{W}\}), \\ 1 \text{ if } b \geq \min\{r, \delta(1+\delta)\overline{W}\}. \end{cases}$$

We note two key features of distribution $F^*(\cdot; r)$. First, this c.d.f. has a mass point at $\min\{r, \delta(1+\delta)\overline{W}\}$. Second, $F^*(\cdot; r)$ has the property that a designated loser is indifferent between submitting a losing bid $\overline{b}$, or placing any bid $b < \overline{b}$ in the support of F^*.

The following result formalizes this discussion:

Proposition 2. *On the equilibrium path, the bidding strategy in any $\sigma \in \Sigma$ that attains $\overline{V}$ is such that, in all periods t, the winning bid is drawn from c.d.f. $F^*(\cdot; r_t)$. Distribution $F^*(\cdot; r_t)$ is degenerate at r_t if $r_t \leq \delta \overline{W}$ and is non-degenerate otherwise, where $\delta \overline{W}$ is the expected continuation payoff of a designated loser under any equilibrium that attains $\overline{V}$.*

Moreover, $\overline{V} > \overline{V}^{CK}$ if and only if $\overline{r} > \delta \overline{W}^{CK}$.

The key takeaways of the results in this section are that a cartel strictly benefits from strategies without common-knowledge bids when the largest point in the support of F_r is high enough; i.e., $\overline{r} > \delta \overline{W}^{CK}$. Moreover, when the reserve price is high enough, the winning bid is drawn from a non-degenerate distribution in any equilibrium that attains $\overline{V}$. Appendix

B shows how $\overline{W}$ is pinned down.

We end this section by noting that our results extend beyond bid-rotation equilibria. Indeed, for any equilibrium with common-knowledge bids such that there is a predetermined winner at each on-path history, we can use the same arguments as in Proposition 2 to construct an equilibrium without common-knowledge bids that delivers greater profits for the cartel. Equilibria with a predetermined winner are natural (and, under certain conditions, optimal) in settings in which firms have heterogenous costs (e.g., costs are drawn i.i.d. each period) and the cartel seeks to allocate projects in a cost-effective manner.

5   Numerical Example

In order to illustrate the features of equilibrium bidding implied by our model, we present a numerical example. For the reserve price, we draw a random variable distributed according to a Cauchy distribution and take its absolute value, i.e., $r_t = |\check{r}_t|$ and $\check{r}_t \sim Cauchy(1)$, where $Cauchy(1)$ is centered around 0 with scale parameter 1.[16] We choose the Cauchy distribution because it has a relatively fat tail. This feature is convenient for illustrating how bidding looks like when there is a very large realization of the reserve price as in the case of the Ohara residences auction. We set the discount factor δ to 0.95.

In the equilibrium with common-knowledge bids that attains $\overline{V}^{CK}$, the winning bid is equal to either r_t or $\overline{W}^{CK}$, whichever is less. When r_t is distributed according to (the positive part of) the Cauchy distribution, $\overline{W}^{CK}$ is about 28.59. This implies that the winning bid b_t equals $\min\{r_t, 27.16\}$ ($28.59 \times 0.95 \approx 27.16$). The top left panel of Figure 2 corresponds to a plot of the reserve price and the winning bid (r_t, b_t) for 200 realizations of r_t. Note that there are 3 realizations of r_t above 27.16 (34.19, 34.94, 62.63) and the corresponding bids are

---

[16]Note that even when the distribution of the reserve price has no mean, expression 1 is well-defined and $\overline{W}^{CK}$ is finite. Likewise, the expression that determines $\overline{V}$ is also well-defined and $\overline{V}$ is finite.
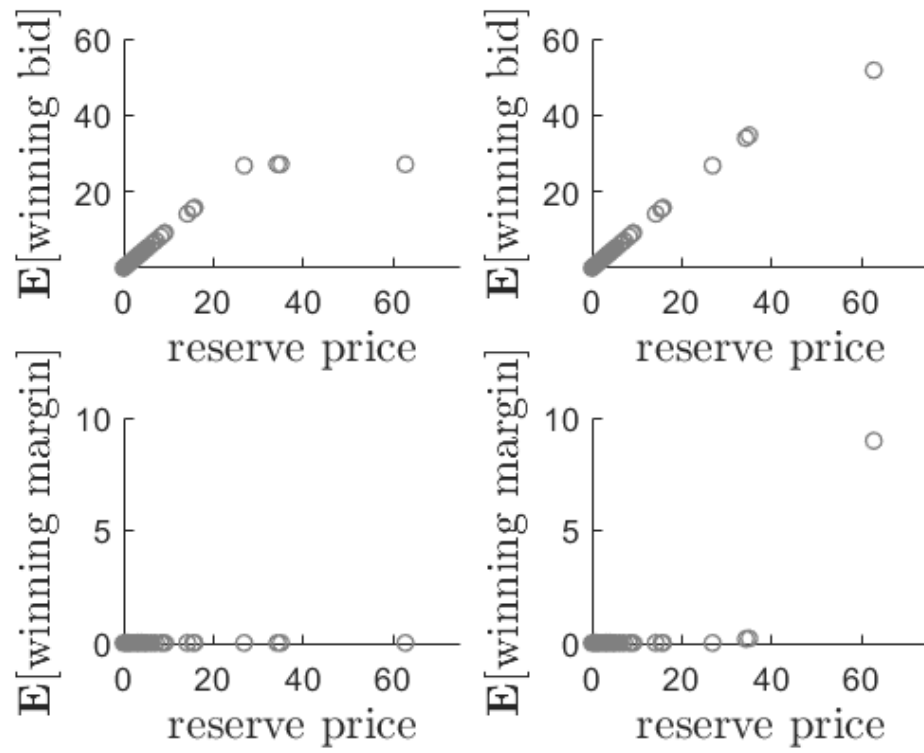
Figure 2: Equilibrium bidding with common-knowledge bids (left panels) and equilibrium without common-knowledge bids (right panels). The top two panels plot the expected winning bid and the bottom two panels plot the expected winning margin.

all capped at 27.16. On the bottom left panel of the figure, we plot the equilibrium winning margin as a function of the reserve price. The winning margin is always zero.

In the equilibrium that attains $\overline{V}$, the winning bid is either equal to r_t or drawn from a non-degenerate distribution $F^*(\cdot; r_t)$ whenever the realization of r_t is above $\delta \times \overline{W}$. The value of $\overline{W}$ under our specification is equal to about 32.78 (and $\delta \times \overline{W}$ is approximately 31.14). For the 3 realizations of r_t that are above 31.14, the winning bid is higher in the equilibrium that attains $\overline{V}$ than in the one that attains $\overline{V}^{CK}$. For the other realizations, the winning bids are the same. The top right panel of Figure 2 illustrates this point. The top right panel plots the reserve price and the expected winning bid. The plot clearly shows that, for the highest realization of the reserve price (r_t =62.63), the expected winning bid is much higher (51.75) than in the top left panel (27.16). In the bottom right panel of Figure 2, we plot the expected winning margin. The winning margin is 0 for 197 realizations and 0.18, 0.20, 8.98 for the three realizations of r_t that exceed 31.14. These patterns are consistent with the bidding patterns of the Kumatori cartel illustrated in Figure 1.

6   Description of the Bidding-Ring in the Court Ruling

In this section, we draw on the court rulings' descriptions of the events leading up to the Ohara Residences auction to provide qualitative evidence that the key theoretical forces we model in Section 4 were present in the Kumatori bidding ring.[17]

According to the Osaka District Court ruling, Mr. Kitagawa of Imakatsu Construction started to believe, around April of 2006, that Tier B firms would be invited to bid on the second part of the Ohara residences project and started mentioning the project at the meetings of the Kumatori Contractors' Cooperative. In a meeting of the Cooperative in late June 2006, Mr. Kitagawa stated that he wanted others to let his firm, Imakatsu Construction,

---

[17]The descriptions are taken from the Osaka District Court ruling (Ruling H21 (Gyo-U) No. 99) and the Osaka High Court ruling (Ruling H24 (Gyo-Ko) No. 101).

win the auction. He also told the members that he would be collecting the detailed project plan that the town distributes at the on-site briefing. This was understood by the members of the cooperative as a preventative measure to make defection more difficult by making cost estimates for other firms harder.[18]

On August 3, 2006, the town office invited five contractors to bid on the housing complex project, including Imakatsu Construction. All invited bidders were Tier B firms and members of the Cooperative. On or around this day, Mr. Kitagawa asked Mr. Nishio to help him with the operation and to obtain confidential information about the project from the town. Mr. Nishio was the vice director of the Cooperative and a senior managing director of Nishinuki Construction at the time. Nishinuki Construction was not one of the invited bidders.

On August 4, 2006, after the general meeting of the Cooperative, Mr. Kitagawa met with the four other bidders that were invited to bid on the Ohara Residences project and repeated his intention to collect the project plans.

The town of Kumatori held an on-site briefing for the Ohara residences project for the five invited firms on August 7. At the on-site briefing, the town distributed the detailed project plan as well as other documents required to estimate costs. These documents were collected immediately after the briefing by an employee of the Cooperative. The plans and the documents were not returned to the bidders until August 21, the day before the auction. During this time, Imakatsu Construction was the only firm that had access to the documents required to estimate costs. After the on-site briefing, Mr. Nishio met with an official of the Buildings Division of the town to obtain information about the engineering estimate.[19]

On August 21, one day before the day of the auction, Mr. Kitagawa and Mr. Nishio met at the office of Imakatsu Construction and decided on a bid of 630 million yen for Imakatsu Construction and bids of above 700 million yen for all other bidders. According to the court

---

[18]Page 18 of the Osaka District Court ruling (Ruling H21 Gyo-U, No. 99).
[19]Page 19 of the Osaka District Court ruling (Ruling H21 Gyo-U, No. 99).

ruling, they decided to set the losing bids to be above 700 million yen so as to make it hard for the other bidders to guess the lowest bid.

Also on the same day, Mr. Nishio contacted the four other invited bidders of the auction. He told them that he would hand them the document containing each firm's break-down of the estimated costs on the day of the auction. Mr. Nishio also told the bidders that he would give instructions on how much to bid on the day of the auction. According to the court ruling, Mr. Nishio's decision to hand the documents and give instructions on bids on the day of the auction (as opposed to two days prior to the day of the auction, as was customary for the bidding ring) was to prevent defection given the large size of the project.[20]

The auction for the public housing complex was held at the town office on August 22 2006. Mr. Nishio went to the town office and stapled together the documents containing the cost break-down of each bidder with the cover page brought by the representatives of the firms. Mr. Nishio also indicated the amount that each bidder should bid by showing a slip of paper with a number above 700 million yen. Importantly, the bid of Imakatsu Construction was kept secret to the other bidders. The representatives of the invited bidders bid the same or slightly above the amount shown on the slip of paper. As a result, Imakatsu Construction won the auction with a bid of 630 million yen. The other bids were 705 million yen, 707 million yen, 710 million yen and 723 million yen, respectively.

7   Discussion

This paper highlights the practical value of randomization in relaxing IC constraints. We also offer a concrete example in which privacy helps cartels obtain higher profits. We conclude the paper with discussions of (i) our model's implications for the price-variance screen for collusion, (ii) how our model explains puzzling bidding patterns documented elsewhere, and

---

[20]Page 4 of the Osaka High Court ruling (Ruling H24 Gyo-Ko, No. 101).

(iii) implications for the information that the auctioneer should make public after each auction.

Price-variance screen. Our model predicts that when the project is relatively small (i.e., r_t is small), the winning bid measured as a fraction of the reserve price ($\frac{\min_i b_{i,t}}{r_t}$) will be close to 1 and the money left on the table will be close to zero. This implies that the time series variance in the winning bids, $\frac{\min_i b_{i,t}}{r_t}$, will be close to zero and the within-auction variance of bids will also be close to zero. These results are consistent with the premise of the price-variance screen. However, when projects are relatively large, our model predicts that there will be considerable time-series variance in the winning bids that results from randomization. Moreover, the within-auction variance of bids will not be close to zero. Hence, increases in the variance of prices should not be taken as failure of collusion when those increases are associated with increases in project sizes.

Explaining other bidding patters. A distinct feature of the equilibrium in Proposition 2 is that the winner has a static incentive to raise its bid whenever the mediator recommends a bid below $\bar{b} = \sup_b \text{supp} \, F^*(\cdot; r)$. This feature is present in the bidding data analyzed in Chassang et al. (2020).[21] Chassang et al. (2020) study the sample distribution of normalized bid differences $\Delta_{i,t} \equiv (b_{i,t} - \wedge b_{-i,t})/r_t$ for procurement auctions let by the Ministry of Land, Infrastructure and Transportation of Japan, and by cities in Ibaraki prefecture and the Tohoku region (bid data from Kumatori are not included in the data analyzed by Chassang et al. (2020)). That paper documents a missing mass in the distribution of Δ around $\Delta = 0$. In other words, winning bids tend to be isolated. This implies that upward deviations by winners are profitable, consistent with the collusive scheme in Proposition 2.

---

[21]Tóth et al. (2014), Imhof et al. (2016) and Clark et al. (2020) document similar patterns. Clark et al. (2020) offer an explanation based on the bidders' desire to leave some margin of error.

What information should the auctioneer make public? Our model assumes that all submitted bids are made public after each auction, consistent with how the town of Kumatori operates. We note that this is crucial for randomized bidding schemes to be profitable. To see why, consider instead a setting in which only the bid of the winning firm is made public. Suppose that the cartel uses bid rotation to allocate contracts, and that the winning bid at some period t is drawn from c.d.f. F_t. Then, for the designated loser not to want to deviate, it must be that

$$\forall b \in \operatorname{supp} F_t, \quad (1 - F_t(b))(b + \delta \times 0) + F_t(b)\delta V^l \leq \delta V^l$$

$$\iff b \leq \delta V^l,$$

where δV^l denotes the loser's expected continuation payoff. Hence, only winning bids below the loser's continuation payoff are sustainable, just as when bids are common-knowledge. Intuitively, when only winning bids are public, a deviation by a designated loser can only be detected if she wins the auction. In contrast, when all bids are made public, a defector can get punished even if she doesn't win.

Appendix

A   Mediation and Solution Concepts

This appendix formally describes the strategies of the mediator, as well as our solution concept.

Mediator's histories and strategies. A period-t history for the mediator,

$$h_{M,t} = \left(r_s, \widehat{\mathbf{b}}_s, \widehat{\gamma}_s, \mathbf{b}_s, \gamma_s\right)_{s < t} \sqcup r_t,$$

records all previous reserve prices, bidding recommendations and realized bids, as well as current reserve price. A strategy $\sigma_M : h_{M,t} \mapsto \Delta(\mathbb{R}_+^2 \times [0,1]^2)$ for the mediator maps mediator histories to a distribution over bidders' bids.

Equilibrium notion. Let $H_{i,t}$ denote the set of period-t histories of player i, and let $H_t = H_{1,t} \times H_{2,t}$. Our solution concept is weak Perfect Public Equilibria (σ, μ), where $\sigma = (\sigma_M, \sigma_1, \sigma_2)$ is a strategy profile, and $\mu = (\mu_1, \mu_2)$ are bidders' beliefs about their opponent's history; i.e., $\mu_i : h_{i,t} \mapsto \Delta(H_t)$, with the property that $\operatorname{supp} \mu_i(h_{i,t}) \subseteq \{h_{i,t}\} \times H_{-i,t}$. In a weak Perfect Public Equilibria (σ, μ), bidders' strategies (σ_1, σ_2) must be sequentially rational at every history, and, for every on-path history $h_{i,t}$, $\mu_i(h_{i,t})$ must be consistent with the mediator's strategy σ_M.[22]

B Proofs

Proof of Proposition 1. Fix an equilibrium $\sigma \in \Sigma^{\mathsf{CK}}$. Let V^w and V^l denote, respectively, the expected discounted payoff that the designated winner and loser at $t = 0$ obtain under σ. Let $\overline{V}^w$ be the highest payoff that the designated winner at $t = 0$ can obtain under an equilibrium in Σ^{CK}. Since $V^w \leq \overline{V}^w$ and $V^l \leq \delta\overline{V}^w$ (since the loser at $t = 0$ is the winner at $t = 1$), $V^w + V^l \leq (1 + \delta)\overline{V}^w$. Since this inequality holds for all $\sigma \in \Sigma^{\mathsf{CK}}$, $\overline{V}^{\mathsf{CK}} \leq (1 + \delta)\overline{V}^w$.

Let $b(r_0)$ be the winning bid at time $t = 0$ under σ. Note that $b(r_0) \leq \delta\overline{V}^w$. Indeed, the continuation payoff of the loser at $t = 0$ can't be larger $\delta\overline{V}^w$ (since the loser at $t = 0$ is the winner at $t = 1$). If $b(r_0) > \delta\overline{V}^w$, the loser would have a strict incentive to undercut $b(r_0)$. Since $b(r_0)$ must be lower than r_0, $b(r_0) \leq \min\{r_0, \delta\overline{V}^w\}$. Hence, $V^w \leq \mathbb{E}_{F_r}[\min\{r_0, \delta\overline{V}^w\}] + \delta^2\overline{V}^w$. Since the inequality holds for all $\sigma \in \Sigma^{\mathsf{CK}}$,

$$\overline{V}^w \leq \frac{1}{1 - \delta^2}\mathbb{E}_{F_r}[\min\{r_0, \delta\overline{V}^w\}].$$

---

[22] The mediator is assumed to be a disinterested third party, with no incentives.

Let $\overline{W}^{\mathsf{CK}}$ be the largest $W \geq 0$ solving

$$W = \frac{1}{1-\delta^2} \mathbb{E}_{F_r}[\min\{r, \delta W\}].$$

Note that $\overline{V}^w \leq \overline{W}^{\mathsf{CK}}$.[23] We now show that $\overline{V}^w = \overline{W}^{\mathsf{CK}}$. Consider the following strategy profile. Along the equilibrium path, in each period t the designated winner bids $\min\{r_0, \delta \overline{W}^{\mathsf{CK}}\}$ and $\gamma = 1$, and the designated loser bids $\min\{r_0, \delta \overline{W}^{\mathsf{CK}}\}$ and $\gamma = 0$. Deviations are punished with Nash reversion. This strategy profile is an equilibrium in Σ^{CK} giving the winner at $t = 0$ an expected discounted payoff of $\overline{W}^{\mathsf{CK}}$. Hence, $\overline{V}^w = \overline{W}^{\mathsf{CK}}$. Since $\overline{V}^{\mathsf{CK}} \leq (1+\delta)\overline{W}^{\mathsf{CK}}$, this equilibrium attains $\overline{V}^{\mathsf{CK}}$. ∎

For each reserve price r and each value $W \geq 0$, let $F(\cdot; r, W)$ be the c.d.f. given as follows. If $\delta W \geq r$, $F(\cdot; r, W)$ puts all its mass at r. If $\delta W < r$, $F(\cdot; r, W)$ is given by:

$$F(b; r, W) = \begin{cases} 0 \text{ if } b < \delta W, \\ 1 - \frac{\delta W}{b} \text{ if } b \in [\delta W, \min\{r, \delta(1+\delta)W\}), \\ 1 \text{ if } b \geq \min\{r, \delta(1+\delta)W\}. \end{cases} \tag{4}$$

Proof of Proposition 2. Fix an equilibrium $\sigma \in \Sigma$. Let V^w and V^l denote, respectively, the expected discounted payoff of the designated winner and loser at $t = 0$ under σ. Let $\hat{V}^w$ be the highest payoff that the winner at time $t = 0$ can obtain under an equilibrium in Σ. By the same arguments as in the proof of Proposition 1, $\overline{V} \leq (1+\delta)\hat{V}^w$.

Let $F(b; r_0)$ be the c.d.f. from which the winning bid at $t = 0$ is drawn under σ. Let $\overline{b} = \sup_b \mathsf{supp}\, F(\cdot; r_0)$ and $\underline{b} = \inf_b \mathsf{supp}\, F(\cdot; r_0)$. Note that the designated loser at $t = 0$ must place a bid weakly higher than $\overline{b}$ under σ; otherwise, it would win with positive probability.

---

[23]Since the right-hand side of (1) is bounded by $\overline{r}/(1-\delta^2)$, $W > \frac{1}{1-\delta^2}\mathbb{E}_{F_r}[\min\{r, \delta W\}]$ for all $W > \overline{W}^{\mathsf{CK}}$.

Moreover, we must have

$$\forall b < \bar{b}, \qquad (1 - F(b; r_0))b \leq \delta \hat{V}^w, \tag{5}$$

$$\forall b \in \text{supp } F(\cdot; r_0), \qquad \bar{b} \leq b + \delta^2 \hat{V}^w. \tag{6}$$

If inequality (5) didn't hold for some $b < \bar{b}$, the loser would have a strict incentive to bid b and win the auction with probability $1 - F(b; r_0)$. If inequality (6) didn't hold for $b \in \text{supp } F(\cdot; r_0)$, the winner would have an incentive to bid $\bar{b} - \epsilon$ with $\epsilon \approx 0$ instead of $b < \bar{b}$. Condition (5) implies

$$\underline{b} \leq \delta \hat{V}^w \text{ and } \forall b < \bar{b}, F_b(b; r_0) \geq 1 - \delta \frac{\hat{V}^w}{b}. \tag{7}$$

Condition (6), together with $\bar{b} \leq r_0$, implies

$$\bar{b} \leq \min\{r_0, \underline{b} + \delta^2 \hat{V}^w\}. \tag{8}$$

Consider the problem of finding the c.d.f. F that maximizes expected winning bid $\int b \, dF$, subject to (7) and (8). When $\delta \hat{V}^w \geq r_0$, the c.d.f. F that solves this problem puts all its mass at r_0. When $\delta \hat{V}^w < r_0$, the c.d.f. F that solves this problem is given by (4), with $r = r_0$ and $W = \hat{V}^w$. Therefore, $V^w \leq \mathbb{E}_{F_r}\left[\int b \, dF(b; r, \hat{V}^w)\right] + \delta^2 \hat{V}^w$. Since the inequality holds for all $\sigma \in \Sigma$,

$$\hat{V}^w \leq \frac{1}{1 - \delta^2} \mathbb{E}_{F_r}\left[\int b \, dF(b; r, \hat{V}^w)\right].$$

Let $\overline{W} \geq 0$ be the largest solution to

$$W = \frac{1}{1 - \delta^2} \mathbb{E}_{F_r}\left[\int b \, dF(b; r, W)\right], \tag{9}$$

30

and note that $\hat{V}^w \le \overline{W}$.[24] For each r, let $F^*(\cdot; r)$ be the c.d.f. given by $F^*(\cdot; r) = F(\cdot; r, \overline{W})$.

We now show that $\hat{V}^w = \overline{W}$. Consider the following strategy profile. Along the equilibrium path, in each period t, after observing r_t the mediator sends bidding recommendation $(\widehat{b}_i, \widehat{\gamma}_i) = (b, 1)$ to the designated winner, with b drawn from c.d.f. $F^*(\cdot; r_t) = F(\cdot; r_t, \overline{W})$; and sends bidding recommendation $(\widehat{b}_j, \widehat{\gamma}_j) = (\bar{b}_t, 0)$ to the designated loser, with $\bar{b}_t = \sup_b \operatorname{supp} F^*(\cdot; r_t)$. Deviations from the mediator's recommendations are punished with Nash reversion.[25] One can check that this strategy profile is an equilibrium in Σ, and gives the designated winner at $t = 0$ a payoff equal to $\overline{W}$. Hence, $\hat{V}^w = \overline{W}$. Since $\overline{V} \le (1 + \delta)\hat{V}^w$, this equilibrium also attains $\overline{V}$.

Finally, note that $\int b\, dF(b; r, W) \ge \min\{r, \delta W\}$ for each r, W, with strict inequality whenever $\delta W < r$.[26] Hence, $\overline{W} \ge \overline{W}^{\mathsf{CK}}$, with strict inequality whenever $\delta \overline{W}^{\mathsf{CK}} < \bar{r}$. Hence, when $\bar{r} > \delta \overline{W}^{\mathsf{CK}}$, we have $(1 + \delta)\overline{W} = \overline{V} > \overline{V}^{\mathsf{CK}} = (1 + \delta)\overline{W}^{\mathsf{CK}}$. ∎

References

ABRANTES-METZ, R., L. FROEB, J. GEWEKE, AND C. TAYLOR (2006): "A variance screen for collusion," *International Journal of Industrial Organization*, 24, 467–486.

ABRANTES-METZ, R., M. KRATEN, A. METZ, AND G. SEOW (2012): "Libor manipulation?" *Journal of Banking & Finance*, 36, 136–150.

---

[24]Since the right-hand side of (9) is bounded, $W > \frac{1}{1-\delta^2}\mathbb{E}_{F_r}\left[\int b\, dF(b; r, W)\right]$ for all $W > \overline{W}$.

[25]Formally, following a deviation, the mediator sends bidding recommendations $(b_i, \gamma_i) = (0, 1)$, which both bidders follow.

[26]When $\delta W \ge r$, $r = \int b\, dF(b; r, W)$. When $\delta W < r$,

$$\delta W < \int b\, dF(b; r, W) = \begin{cases} \delta W[1 + \ln(1 + \delta)] & \text{if } \delta(1 + \delta)W < r, \\ \delta W\left[1 + \ln(r) - \ln(\delta W)\right] & \text{if } \delta(1 + \delta)W \ge r. \end{cases}$$

ASKER, J. (2010): "A study of the internal organization of a bidding cartel," *The American Economic Review*, 724–762.

ATHEY, S. AND K. BAGWELL (2001): "Optimal collusion with private information," *RAND Journal of Economics*, 32, 428–465.

——— (2008): "Collusion with persistent cost shocks," *Econometrica*, 76, 493–540.

ATHEY, S., K. BAGWELL, AND C. SANCHIRICO (2004): "Collusion and price rigidity," *The Review of Economic Studies*, 71, 317–349.

ATHEY, S., J. LEVIN, AND E. SEIRA (2011): "Comparing open and Sealed Bid Auctions: Evidence from Timber Auctions*," *The Quarterly Journal of Economics*, 126, 207–257.

AWAYA, Y. AND V. KRISHNA (2016): "On Communication and Collusion," *American Economic Review*, 106, 285–315.

BAJARI, P. AND L. YE (2003): "Deciding between competition and collusion," *Review of Economics and Statistics*, 85, 971–989.

BALDWIN, L. H., R. C. MARSHALL, AND J.-F. RICHARD (1997): "Bidder collusion at forest service timber sales," *Journal of Political Economy*, 105, 657–699.

BERNHEIM, D. AND E. MADSEN (2017): "Price Cutting and Business Stealing in Imperfect Cartels," *American Economic Review*, 107, 387–424.

BORENSTEIN, S. AND A. SHEPARD (1996): "Dynamic Pricing in Retail Gasoline Markets," *The RAND Journal of Economics*, 27, 429–451.

CARLTON, D. (1986): "The Rigidity of Prices," *The American Economic Review*, 76, 637–658.

CHASSANG, S., K. KAWAI, J. NAKABAYASHI, AND J. ORTNER (2020): "Robust Screens for Non-Competitive Bidding in Procurement Auctions," .

CHASSANG, S. AND G. P. I. MIQUEL (2019): "Crime, intimidation, and whistleblowing: A theory of inference from unverifiable reports," *The Review of Economic Studies*, 86, 2530–2553.

CHASSANG, S. AND J. ORTNER (2019): "Collusion in Auctions with Constrained Bids: Theory and Evidence from Public Procurement," *Journal of Political Economy*, 127, 2269–2300.

CHIAPPORI, P.-A., S. LEVITT, AND T. GROSECLOSE (2002): "Testing mixed-strategy equilibria when players are heterogeneous: The case of penalty kicks in soccer," *American Economic Review*, 92, 1138–1151.

CLARK, R., D. COVIELLO, J.-F. GAUTHIER, AND A. SHNEYEROV (2018): "Bid rigging and entry deterrence in public procurement: Evidence from an investigation into collusion and corruption in Quebec," *The Journal of Law, Economics, and Organization*, 34, 301–363.

CLARK, R., D. COVIELLO, AND A. D. LEVERANO (2020): "Complementary Bidding and the Collusive Arrangement: Evidence from an Antitrust Investigation," .

COMPTE, O. (1998): "Communication in repeated games with imperfect private monitoring," *Econometrica*, 597–626.

CONDORELLI, D. AND B. SZENTES (2020): "Information Design in the Holdup Problem," *Journal of Political Economy*, 128, 681–709.

CONLEY, T. G. AND F. DECAROLIS (2016): "Detecting bidders groups in collusive auctions," *American Economic Journal: Microeconomics*, 8, 1–38.

CREDE, C. (2019): "A Structural Break Cartel Screen for Dating and Detecting Collusion," *Review of Industrial Organization*, 54, 543—574.

EDERER, F., R. HOLDEN, AND M. MEYER (2018): "Gaming and strategic opacity in incentive provision," *The RAND Journal of Economics*, 49, 819–854.

ELLISON, G. (1994): "Theories of cartel stability and the joint executive committee," *The Rand journal of economics*, 37–57.

GREEN, E. J. AND R. H. PORTER (1984): "Noncooperative collusion under imperfect price information," *Econometrica: Journal of the Econometric Society*, 87–100.

HALTIWANGER, J. AND J. HARRINGTON (1991): "The Impact of Cyclical Demand Movements on Collusive Behavior," *The RAND Journal of Economics*, 22, 89–106.

HARRINGTON, J. E. (2008): "Detecting cartels," *Handbook of antitrust economics*, 213, 245.

HARRINGTON, J. E. AND A. SKRZYPACZ (2011): "Private monitoring and communication in cartels: Explaining recent collusive practices," *The American Economic Review*, 101, 2425–2449.

HENDRICKS, K. AND R. PORTER (1988): "An Empirical Study of an Auction with Asymmetric Information," *American Economic Review*, 78, 865–883.

IMHOF, D., Y. KARAGÖK, AND S. RUTZ (2016): "Screening for bid-rigging-does it work?" Tech. rep., Université de Fribourg.

JEHIEL, P. (2015): "On transparency in organizations," *The Review of Economic Studies*, 82, 736–761.

JIMÉNEZ, J. L. AND J. PERDIGUERO (2012): "Does Rigidity of Prices Hide Collusion," *International Journal of Industrial Organization*, 41, 223–248.

KAMENICA, E. AND M. GENTZKOW (2011): "Bayesian persuasion," *American Economic Review*, 101, 2590–2615.

KANDORI, M. (2003): "Randomization, communication, and efficiency in repeated games with imperfect public monitoring," *Econometrica*, 71, 345–353.

KANDORI, M. AND H. MATSUSHIMA (1998): "Private observation, communication and collusion," *Econometrica*, 627–652.

KANDORI, M. AND I. OBARA (2006): "Efficiency in repeated games revisited: The role of private strategies," *Econometrica*, 74, 499–519.

KAWAI, K. AND J. NAKABAYASHI (2018): "Detecting Large-Scale Collusion in Procurement Auctions," *Available at SSRN 2467175*.

LEVENSTEIN, M. (1997): "Price Wars and the Stability of Collusion: A Study of the Pre-World War I Bromine Industry," *The Journal of Industrial Economics*, 45, 117–137.

MASKIN, E. AND J. TIROLE (1988): "A theory of dynamic oligopoly, II: Price competition, kinked demand curves, and Edgeworth cycles," *Econometrica: Journal of the Econometric Society*, 571–599.

OHASHI, H. (2009): "Effects of transparency in procurement practices on government expenditure: A case study of municipal public works," *Review of Industrial Organization*, 34, 267–285.

ORTNER, J. AND S. CHASSANG (2018): "Making corruption harder: Asymmetric information, collusion, and crime," *Journal of Political Economy*, 126, 2108–2133.

PEREZ-RICHET, E. AND V. SKRETA (2018): "Test design under falsification," *Work. Pap., Sci. Po, Paris Access provided by University of Chicago Libraries on*, 9, 04–19.

PESENDORFER, M. (2000): "A study of collusion in first-price auctions," *The Review of Economic Studies*, 67, 381–411.

PORTER, R. (2005): "Detecting Collusion," *Review of Industrial Organization*, 26, 147–167.

PORTER, R. H. (1983): "A study of cartel stability: the Joint Executive Committee, 1880-1886," *The Bell Journal of Economics*, 301–314.

PORTER, R. H. AND J. D. ZONA (1993): "Detection of Bid Rigging in Procurement Auctions," *Journal of Political Economy*, 101, 518–38.

——— (1999): "Ohio School Milk Markets: An Analysis of Bidding," *RAND Journal of Economics*, 30, 263–288.

RAHMAN, D. (2012): "But who will monitor the monitor?" *American Economic Review*, 102, 2767–97.

——— (2014): "The power of communication," *American Economic Review*, 104, 3737–51.

RAHMAN, D. AND I. OBARA (2010): "Mediated partnerships," *Econometrica*, 78, 285–308.

ROESLER, A.-K. AND B. SZENTES (2017): "Buyer-optimal learning and monopoly pricing," *American Economic Review*, 107, 2072–80.

ROTEMBERG, J. J. AND G. SALONER (1986): "A supergame-theoretic model of price wars during booms," *The American Economic Review*, 390–407.

SCHURTER, K. (2017): "Identification and inference in first-price auctions with collusion," Tech. rep., working Paper, University of Chicago.

SKRZYPACZ, A. AND H. HOPENHAYN (2004): "Tacit collusion in repeated auctions," *Journal of Economic Theory*, 114, 153–169.

SUGAYA, T. AND A. WOLITZKY (2017): "Bounding equilibrium payoffs in repeated games with private monitoring," *Theoretical Economics*, 12, 691–729.

——— (2018): "Maintaining privacy in cartels," *Journal of Political Economy*, 126, 2569–2607.

TÓTH, B., M. FAZEKAS, Á. CZIBIK, AND I. J. TÓTH (2014): "Toolkit for detecting collusive bidding in public procurement," *With examples from Hungary.*

WALKER, M. AND J. WOODERS (2001): "Minimax Play at Wimbledon," *American Economic Review*, 91, 1521–1538.

WANG, Z. (2009): "(Mixed) Strategy in Oligopoly Pricing: Evidence from Gasoline Price Cycles Before and Under a Timing Regulation," *Journal of Political Economy*, 117, 987–1030.