# A Merger Paradox： Proposal Right and Price Discrimination 

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## A (Horizontal) Merger Paradox

Is a horizontal merger (of sellers) always beneficial?

- No, shown by Salant et al. (1983, QJE).
$\because$ The price $\nearrow$, but the share $\searrow$ of merging firms.
- In a Cournot oligopoly with constant marginal cost c,
- a merger is beneficial if over $80 \%$ of firms merge in general. By contrast, the merger to monopoly seems to be always beneficial.
- The merged monopolist is able to charge the monopoly price,
- by keeping the market share (one).

However, we find it is not true

- if the sellers' vertical bargaining power $\beta$ over buyers is weak.


## Vertical Bargaining Power?

Sellers earn $\beta$ of total surplus as a generalized Nash bargaining solution.

- It is supported by the ultimatum bargaining with random proposers.
- In this procedure, bargaining power $\beta \in[0,1]$ is modeled as
- the probability $\beta$ of being proposers.
$\beta=1$ : Sellers are proposers (or price-makers) w.p. 1;
- Each seller is able to offer a price (contract) to every buyer.
$\beta=0$ : Sellers are responders (or price-takers) w.p. 1;
- Each seller decides to accept/reject contracts offered by buyers.

When $\beta \approx 0$, only after sellers' merger (not buyers' merger)

- buyers can extract more surplus by using discriminated prices.
$\rightarrow$ Sellers' merger to monopoly reduces sellers' surplus.


## Price Discrimination

Suppose buyers are chosen as proposers. Then,

- they attempt to extract surplus by offering non-liner prices to sellers. Sellers' merger facilitates such price discrimination by buyers:
- Before the merger, it is hard to implement non-linear prices.
$\because$ A low-cost seller $j$ has power to switch from a buyer to another because other buyers want to buy a unit from $j$.
$\rightarrow$ All buyers cannot offer a price lower than the Walrasian to $j$.
- After the merger, all sellers behave as a single entity.
$\rightarrow$ Low cost $j$ cannot switch independently (i.e. no power to switch) even if $j$ faces a lower price, and it harms $j$.

Therefore, buyers can impose non-linear prices only if sellers' power $\beta$ is low after the merger.

## Buyer Power in Practice

Practically also，the regulators take such power into consideration． ［US FTC guideline（p．27）］：Powerful buyers are often able to negotiate fa－ vorable terms with their suppliers．Such terms may reflect the lower costs of serving these buyers，but they also can reflect price discrimination in their favor．The Agencies consider the possibility that powerful buyers may con－ strain the ability of the merging parties to raise prices．．．

Also in EU and in Japan：

```
(5) 需要者からの競争圧力
当該一定の取引分野への競争圧力は，次の取引段階に位置する需要者側から生じることもある。需要者 が，当事会社グループに対して，対抗的な交渉力を有している場合には，取引関係を通じて，当事会社グ ループがある程度自由に価格等を左右することをある程度妨げる要因となり得る。需要者側からの競争圧力が働いているか否かについては，次のような需要者と当事会社の取引関係等に係る状況を考慮する。
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## Remarks

The merger is not always harmful even if $\beta \approx 0$.

- The merger is harmful only if the substitutability of sellers' products is high (e.g. homogeneous prodcusts, or perfect substitutes).
- By contrast, if sellers' products are complements for buyers, the merger is neutral even when they have no power.
- The separability of efficient allocation plays a key role. We also assume that
- No entry: No new buyers/sellers enter after the merger.
- No synergy: The merger does not decrease the cost.
- Fixed $\beta$ : The merger does not enhance their power largely.
- Only in the talk. The paper shows the result holds as long as sellers' merger does not raise their power $\beta$ significantly.


## Literature Review

A merger paradox is repeatedly examined.

- A partial merger in oligopoly.
- Cournot: Salant et al. (1983), Farrel \& Shapiro (1990),...
- Bertrand with product differentiation: Deneckere \& Davidson (1985),...
- A merger to monopoly under information asymmetry.
- Waehrer and Perry (2003), Froeb et al. (2016), Loertscher and Marx (2019a,b,c).
- The merger reduces sellers' information rent.
- A merger when each seller can supplies only a specific buyer.
- Iozzi and Valleti (2014).
- When sellers are locked-in, the merger always benefits sellers.

Our finding gives a new reasoning for merger paradox:

- caused by buyers' price discrimination when sellers' power is weak.


## Short Summary

Our research question is:

- Is sellers' merger to monopoly always beneficial to them?

We show that

- NO, even the merger to monopoly is harmful, if
- their vertical bargaining power is weak,
+ their products are substitutes,
+ the merger does not increase the power largely.
Intuitively, in such a case,
- buyers (proposers) can exploit a larger amount of surplus
- by offering discriminated prices only after the merger.
* The same result hold for buyers:
- If sellers are proposers, buyers' merger is harmful.


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## A $2 \times 2$ Example: Pre-Merger Outcome

Suppose $\beta=0$ (buyers are proposers).

- two buyers ( $i=A, B$ ) and
- two sellers ( $j=C, D$ ) exist,
- $C, D$ produce same products.
$\rightarrow$ An efficient allocation

$$
\left(x_{C}^{A}, x_{D}^{A}, x_{C}^{B}, x_{D}^{B}\right)=(1,0,0,1) .
$$

| $x$ | $v^{A}$ | $v^{B}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 9 |  | $x$ | $c_{C}$ | $c_{D}$ |
| 2 | 17 | 15 |  | 1 | 1 |  |
| 2 | 9 | 10 |  |  |  |  |

Table: Valuations $v^{A}, v^{B}$ and costs $c_{C}, c_{D}$ in Example.

Before the merger, no price discrimination emerges.

- $C \& D$ supply one unit independently at the Walrasian price $p=7$.
- Payoffs $u^{A}=3, u^{B}=2, u_{C}=6, u_{D}=6$.
- Supported by eqm price offers: $p_{j}^{i}=7$ for $i=A, B, j=C, D$.
$\rightarrow$ The pre-merger sellers' joint profit is $6+6=12$.
Since C,D supplies one unit, non-linear pricing is not available for A,B.


## A $2 \times 2$ Example: Post-Merger Outcome

$C, D$ are merged to $J$.

- $\operatorname{Cost} c_{J}(x)=\min \left[c_{C}(z)+c_{D}\left(z^{\prime}\right)\right]$ s.t. $x=z+z^{\prime}$.
$\rightarrow$ An efficient allocation

$$
\left(x_{J}^{A}, x_{J}^{B}\right)=(1,1) .
$$

| $x$ | $v^{A}$ | $v^{B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 9 |
| 2 | 17 | 15 |$\quad$| $x$ | $c_{C}$ | $c_{D}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 |
| 2 | 9 | 10 |

Table: Valuations $v^{A}, v^{B}$ and costs
$c_{C}, c_{D}$ in Example.
After the merger, buyers can offer discriminatory prices.

- $A$ offers $p=6$ for a first unit, 7 for a second $\left(p^{A}=(6,7)\right.$ ).
- $B$ offers $p=7$ for a first unit, 6 for a second $\left(p^{B}=(7,6)\right)$.
- These offers constitute eqm and yield the buyer-optimal core.
$\rightarrow$ By $u^{\prime A}=4>u^{A}, u^{\prime B}=2=u^{B}$, the post-merger sellers' joint profit is reduced to 11 (from 12).

Since $J$ supplies two units, non-linear pricing is available for $A, B$.

## A $2 \times 2$ Example: Price Discrimination

Before the Merger :

- The unit Walrasian price $p_{j}^{i}=7$ for all pairs $(i, j)$.
- It is impossible to implement personalized prices.
e.g. Suppose buyer $A$ deviates to reduce $p_{C}^{A^{\prime}}=6.5$ to $C$.
$\rightarrow$ Low-cost seller $C$ switches from buyer $A$ to $B\left(\because p_{C}^{B}=7>6.5\right)$. However, after the Merger :
- Each buyer offers personalized \& non-linear pricing.
- $A$ offers non-linear $p^{A}=(6,7)$, and $B$ offers $p^{B}=(7,6)$.
- Since every choice $((2,0),(1,1),(0,2))$ yields profit 13 , these are indifferent for the merged $C D$ supplying two units.

Only after the merger, the price discrimination is possible.

- The merger makes the joint deviation of ( $B, C$ ) impossible.
- After the merger, sellers $C, D$ cannot deviate separately.


## An Example of Neutral Merger

Sellers' products are differentiated.

- Those are complements.
- The efficient allocation is

$$
\because\left(x_{C}^{A}, x_{D}^{A}, x_{C}^{B}, x_{D}^{B}\right)=(1,1,1,1) .
$$

| $\underline{x}$ | $v^{A}$ | $v^{B}$ |  | $x_{j}$ | $c_{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{D}$ |  |  |  |  |  |
| 1 | 10 | 9 |  | $c_{D}$ | 1 |
|  | 1 |  |  |  |  |
| 2 | 15 | 13 |  | 2 | 3 |

Table: $\underline{x}=\min \left\{x_{C}, x_{D}\right\}$.

After the merger, each buyer offers $p=5$ for each bundle of $(C, D)$. $\rightarrow$ Profits are $\left(u^{A}, u^{B}, u_{C D}\right)=(10-5,9-5,5+5-3-4)=(5,4,3)$.

Even before the merger, sellers' joint profit is the same.

- Unit prices are $p_{C}^{A}=p_{D}^{A}=(5 / 2,5 / 2)$ and $p_{C}^{B}=p_{D}^{B}=(5 / 2,5 / 2)$.
- $\left(u^{A}, u^{B}, u_{C}, u_{D}\right)=(5,4,3 / 2,3 / 2)$, i.e. $u_{C}+u_{D}=u_{C D}=3$.
e.g. If $A$ reduces $p_{C}^{A}<5 / 2, C$ deviates to supply two units to $B$ only.
- The joint deviation of $A C$ is meaningless by complementarity.


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## Timeline of the Merger Game

(1) Sellers decide to merge or not to merge.

- If merged, monopolistic entity $J$ has cost function $C_{J}$.
(2) The vertical bargaining takes place.
- Prices are determined by the (generalized) Nash bargaining solution (NBS) $u^{*}=\beta \psi+(1-\beta) \phi$.
- $\phi$ is the (first price package) auction outcome where buyers are proposers.
- $\psi$ is the reverse auction outcome where sellers are proposers.

By backward induction, we first solve the vertical bargaining.

- We only consider the auction when sellers are not merged.
- However, everything is parallel for the case of merger.


## Model

An allocation $z=\left(z_{1}^{1}, \ldots, z_{s}^{b}\right)$.

- each $z_{j}^{i} \in \mathbb{R}_{+}^{N}$ is a bundle that $j$ sells to $i$.

Buyers $I$, indexed by $i=1, \ldots, b$.

- $i^{\prime}$ S Profit $\Pi^{i}(\tau, z)=v^{i}\left(\sum_{j} z_{j}^{i}\right)-\sum_{j} \tau_{j}^{i}\left(z_{j}^{i}\right)$.
- Valuation $v^{i}$ is increasing, concave.

Sellers $J$, indexed by $j=1, \ldots, s$.

- $j^{\prime}$ 's Profit $\Pi_{j}(\tau, z)=\sum_{i} \tau_{j}^{i}\left(z_{j}^{i}\right)-c_{j}\left(\sum_{i} z_{j}^{i}\right)$.
- Cost $c_{j}$ is increasing, convex.

Here, $\tau_{j}^{i}(z)$ is a payment function from $i$ to $j$.

- $\tau_{j}^{i}$ (contract) can be non-linear, personalized.
- If $\tau_{j}^{i}$ is linear, $\tau_{j}^{i}(z)=\tau_{j}^{i} \cdot z$.
- If $\tau_{j}^{i}$ is non-personalized, $\tau_{j}^{i}(z)=\tau(z)$.


## First Price (Pay-as-Bid) Package Auction

Suppose buyers are chosen as proposers. Then,
1 Each buyer $i$ offers menus $\left(\tau_{1}^{i}, \ldots, \tau_{s}^{i}\right)$ to all sellers, simultaneously.
2 Given $\left(\tau_{j}^{1}, \ldots, \tau_{j}^{b}\right)$, each seller $j$ (or J) chooses $z_{j}=\left(z_{j}^{1}, \ldots, z_{j}^{b}\right)$.
3 The allocations $z$ and the payments $\tau(z)$ are enforced.
This is a contracting game with multi-principals and multi-agents.

- Or, it is the pay-as-bid (first price) combinatorial auction.

Given $\tau^{-i}$, each buyer $i$ solves:

$$
\begin{align*}
& \max _{\tau^{i}=\left(\tau_{1}^{i}, \ldots, \tau_{s}^{i}\right)} v^{i}\left(\sum_{j} z_{j}^{i}\right)-\sum_{j} \tau_{j}^{i}\left(z_{j}^{i}\right) \\
& \text { s.t. } z_{j}=\left(z_{j}^{1}, \ldots, z_{j}^{b}\right) \in \arg \max \sum_{i} \tau_{j}^{i}\left(z_{j}^{i}\right)-c_{j}\left(\sum_{i} z_{j}^{i}\right) \tag{j}
\end{align*}
$$

## Equilibrium of the Auction

Since there are many Nash equilibria, we take an appealing class of it.

- A profile $\tau=\left(\tau^{1}, \ldots, \tau^{b}\right)$ is a profit-target NE if
- $\tau$ is a Nash eqm. (NE) .
- Each $\tau_{j}^{i}(z)=\max \left\{\left[v^{i}\left(z+\sum_{l<j} y_{l}^{i}\right)-v^{i}\left(\sum_{l<j} y_{l}^{i}\right)\right]-\pi_{j}^{i}, 0\right\}$.
- $\tau_{j}^{i}$ implements target bundle $y_{j}^{i} \&$ profit $\pi_{j}^{i}$.
- Profit-target NE $\tau$ is a truthful eqm because $\sum_{j} \tau_{j}^{i}(z)=v^{i}(z)-\sum_{j} \pi_{j}^{i}$.
- $\ln \tau$, each $i$ earns the sum of target profits, $\sum_{j} \pi_{j}^{i}$.

The profit-target NE payoffs are characterized by the core.

## Theorem (Shirata (2017))

The set of profit-target NE payoffs is equal to the buyer-optimal core (BOC).

- An extension of the menu auction (Bernheim \& Whinston (1986)).
- If sellers are bidders, it is the seller-optimal core.


## Buyer-optimal Core

We define the value of coalition $S \subset M=I \cup J$ as

$$
w(S)=\max _{z \in F(S)} \sum_{i \in S} v^{i}\left(\sum_{j \in S} z_{j}^{i}\right)-\sum_{j \in S} c_{j}\left(\sum_{i \in S} z_{j}^{i}\right),
$$

By letting $u$ be a payoff vector, the core of $(M, w)$ is

$$
\begin{aligned}
& \operatorname{Core}(M, w)=\left\{u \mid \sum_{i \in I} u^{i}+\sum_{j \in J} u_{j} \leq w(M)\right\} \text { (Feasibility) } \\
& \cap\left\{u \mid \sum_{i \in I \cap S} u^{i}+\sum_{j \in J \cap S} u_{j} \geq w(S) \forall S \subseteq M\right\} \text { (No blocking coalition). }
\end{aligned}
$$

Every $\phi$ in the buyer-optimal core is on its Pareto-frontier for buyers.

- $\nexists u \in \operatorname{Core}(M, w)$ such that $u^{i} \geq \phi^{i}$ (strict for some $i \in I$ ).
- (The seller-optimal core is its Pareto-frontier for sellers.)


## A generalized Nash Bargaining Solution

Consider the bargaining problem ( $U, d$ ) between buyers and sellers.

- $U$ is feasible if $\sum_{i} u^{i}+\sum_{j} u_{j} \leq w(M)$.
- The disagreement point $d_{I}$ of buyers is $\Psi^{B}=\sum_{i} \psi^{i}$.
- If some seller $j$ earns more than its seller-optimal $\psi_{j}$, buyers deviate jointly with other sellers and the bargaining breaks down.
- Also, the disagreement point $d_{J}$ of sellers is $\Phi_{S}=\sum_{j} \phi_{j}$. The solution $u^{*}$ is given by the maximizer of Nash product $\left(U^{B}-\Psi^{B}\right)^{1-\beta}\left(U_{S}-\Phi_{S}\right)^{\beta}$, where $U^{B}=\sum_{i} u^{i}, U_{S}=\sum_{j} u_{j}$.


## Proposition

The bargaining solution $u^{*}=(1-\beta) \phi+\beta \psi$ with power $\beta \in[0,1]$.
Thus, $u^{*}$ is the expected outcome of the two auctions.

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## Timeline of the Merger Game

Recall the timeline.
(1) Sellers decide to merge or not to merge.

- If merged, monopolistic entity $J$ has cost function $C_{J}$.
(2) The vertical bargaining takes place.
- The solution $u^{*}=(1-\beta) \phi+\beta \psi$ is realized.

We solved Stage 2. Now, go back to Stage 1.

- If merged, the monopolistic seller J's cost function is
- $C_{J}(z)=\min _{\left(z_{j}\right)_{j \in J}} \sum_{j \in J} c_{j}\left(\sum_{i \in I} z_{j}^{i}\right)$ s.t. $\sum_{i, j} z_{j}^{i}=z$ (no synergy).
- We only solve the two extreme cases, $\beta=0,1$.

The other cases of $\beta \in(0,1)$ is the linear combination of them.

## $\beta=1$ : The Merger Benefit of Proposers

When sellers have full power $(\beta=1)$, solution $u^{*}$ is the seller-optimal $\psi$.

- When not merged, sellers (bidders) compete in price in the reverse auction.
- Since the merger removes their competition, the merged entity $J$ extracts full surplus by offering perfectly discriminated prices.
$\rightarrow$ Sellers' merger always raises their joint profit.


## Proposition

If $\beta=1$, then sellers' merger is always beneficial to them.
After the merger, and sellers' joint profit $\Psi_{J, S}=w(M)$ and buyers obtain zero surplus.

## The Example Revisited

Before the merger,

- seller $j$ offers a unit price $p_{j}^{i}=9$.
$\because p=9$ is the maximal Walrasian.

| $x$ | $v^{A}$ | $v^{B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 9 |
| 2 | 17 | 15 |$\quad$| $x$ |
| :---: |$c_{C} \quad c_{D}$

- $\left(\phi^{A}, \phi^{B}, \phi_{C}, \phi_{D}\right)=(1,0,8,8)$.

After the merger, the merged entity extract full surplus by using discriminated prices.

- The merged $J$ offers $p^{A}=(10,7)$ and $p^{B}=(9,6)$.
- Theses are personalized and non-linear.
$\rightarrow$ The merged $J$ earns full surplus $w(A, B, C, D)=17$.
- The joint profit increases; $\Psi_{\text {Pre }, S}=16 \nearrow \Psi_{J, S}=w(M)=17$.


## $\beta=0$ : The Merger Harm of Responders

When sellers have no power $(\beta=0)$, solution $u^{*}$ is the buyer-optimal $\phi$.

## Proposition

Suppose $\beta=0$. Then, sellers' merger is never beneficial to them. Furthermore, generically:

- If there is a separable core allocation, the merger is harmful.
- Otherwise, the merger is neutral.

A core allocation $x$ is separable if there exists a set $\mathcal{S}$ such that
(1) $\mathcal{S}$ is a partition of $M=I \cup J$,
(2) $x_{j}^{i}>0$ whenever $i, j \in S$ for some $S \in \mathcal{S}$, and $x_{j}^{i}=0$ otherwise. All trades are closed in each coalition $S \in \mathcal{S}$.

## The Example Revisited

Before the merger,

- buyer $i$ offers a unit price $p_{j}^{i}=7$.
$\because p=7$ is the minimal Walrasian.

| $x$ | $v^{A}$ | $v^{B}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 9 |  |  |  |
| 2 | 17 | 15 |  |  | $x$ |
| $c_{C}$ | $c_{D}$ |  |  |  |  |
|  | 1 | 1 | 1 |  |  |
| 2 | 10 |  |  |  |  |

- $\left(u^{A}, \phi^{B}, \phi_{C}, \phi_{D}\right)=(3,2,6,6)$.

Sellers' merger strictly reduces their joint profit to $\Phi_{J, S}=11$.
$\because$ A core allocation is separable:

- $\mathcal{S}=\{A C, B D\}$ by $\left(x_{C}^{A}, x_{D}^{A}, x_{C}^{B}, x_{D}^{B}\right)=(1,0,0,1)$.

After the merger, the joint deviation of $S \in \mathcal{S}$ is impossible.

- Every IC constraint for $S \in \mathcal{S}$ is binding before the merger.
- The merger removes those biding constraints.
- If $\phi^{A}+\phi_{C}=w(A C)=9, \phi^{B}+\phi_{C}=w(B C)=8$ is violated in $\left(\phi^{A}, \phi^{B}, \phi_{J}\right)=(4,2,11) \rightarrow \mathrm{BC}$ can deviate only before the merger.


## A Sketch of the Proof

If $\beta=0$, the auction outcome is buyer-optimal.

- Maximize the joint profit of buyers, $\Phi^{B}=\sum_{i=1}^{b} \phi^{i}$,
- subject to no-blocking-coalition conditions (+ feasibility).
- the value of coalitional deviation $w(S) \leq \sum_{i \in S} \phi^{i}+\sum_{j \in S} \phi_{j}$.
$\Longleftrightarrow w(S) \leq w(M)-\left[\sum_{i \notin S} \phi^{i}+\sum_{j \notin S} \phi_{j}\right]$,
(by feasibility $w(M)=\sum_{i \in I} \phi^{i}+\sum_{j \in J} \phi_{j}$ ).
- Before the merger,
- any coalitional deviation of $S \subset M$ is possible.
- After the merger,
- only a coalitional deviation of $S \supset J$ including all sellers is possible.
$\rightarrow$ The merger reduces possible coalitional deviations.


## The Merger Eliminates Constraints

Pre-merger problem:

$$
\begin{array}{ccc}
\max _{\phi} U^{B}=\sum_{i \in I} u^{i} & \text { (Pre) } & \max _{\phi} U^{B}=\sum_{i \in I} \phi^{i} \\
\text { s.t. } w(M)-\sum_{i \notin S} u^{i} \geq w(S) & \text { s.t. } w(M)-\sum_{i \notin S} u^{i} \geq w(S) \\
\text { for any } S \text { with } S \supseteq J & \text { (2) } & \text { for any } S \text { with } S \supseteq J . \\
w(M)-\sum_{i \notin S} u^{i}-\sum_{j \notin S} u_{j} \geq w(S) & \\
\text { for any } S \text { with } S \nsupseteq J & \text { (3) } & \left(u^{i}, u_{j} \geq 0 \text { for any } i, j, S\right) .
\end{array}
$$

- The constraints (3) is removed in (PoS).
- Thus, solution $\Phi_{\text {Pre }}^{B} \leq \Phi_{J}^{B}$ for proposers (buyers).
$\rightarrow$ By $\beta=0$, sellers' merger is not beneficial to sellers.


## When are the Constraints Binding?

If the constraints (3) is not binding,

- the outcome in (PoS) is the same as in (Pre), i.e. $\Phi_{\text {Pre }}^{B}=\Phi_{S}^{B}$.
- The merger is neutral.

However, $\Phi_{\text {Pre }}^{B} \lesseqgtr \Phi_{S}^{B}$, if at least one of them is binding.

- Sellers' merger raises buyers' joint profit.
- Since $w(M)$ is irrelevant to the merger, sellers' joint profit is reduced by the merger.
- The separability implies the bindingness of constraints in (3) whenever the binding constraints are linearly independent (genericity).


## The Separability and Bindingness

The allocation $x$ is separated by partition $\mathcal{S}$ of $M$

- if $x_{j}^{i}>0$ for $i, j \in S \in \mathcal{S} \& x_{j}^{i}=0 \mathrm{o} / \mathrm{w}$.

If the allocation is separated by $\mathcal{S}$,

- welfare $w$ is additively separable in $\mathcal{S}, w(M)=\sum_{S \in \mathcal{S}} w(S)$.

For example $\mathcal{S}=\left(S_{1}, S_{2}\right)$ \& the constraint (3) is not binding for $S_{1}$,

$$
\text { IC } w\left(S_{1}\right)<w(M)-\sum_{i, j \in S_{2}}\left(u^{i}+u_{j}\right) \Longleftrightarrow w\left(S_{2}\right)>\sum_{i, j \in S_{2}}\left(u^{i}+u_{j}\right)
$$

$\rightarrow$ There is a profitable deviation for $S_{2}$.

- Thus, the constraints for $S_{1}$ and for $S_{2}$ are binding in the NE.

Since sellers' merger removes those binding constraints,

- $\Phi^{B}$ is increased $\& \Phi_{S}$ is decreased by the merger.
- It is achieved by buyers' price discrimination.


## $\beta \in(0,1):$ The Result

Let $\bar{\beta}_{J}$ be such that $\frac{1-\bar{\beta}_{J}}{\bar{\beta}_{J}}=\frac{\Phi_{\mathrm{Pre}, S} S^{-} \Phi_{J, S}}{\Psi_{\mathrm{Pre}}^{B}}=\frac{\Phi_{S}^{B}}{\Psi_{\mathrm{Pre}}^{B}}-1$, where $\Phi$ is BOC \& $\Psi$ is SOC.

## Proposition

- When there is a separable core allocation, sellers' merger is harmful to them if $\beta<\bar{\beta}_{J}$, and it is beneficial to them if $\beta>\bar{\beta}_{J}$.
- Otherwise, sellers' merger is beneficial for any $\beta \in(0,1)$.
- In Example 1, by letting $\mathcal{S}=\{A C, B D\}$, the allocation is separable.
$\rightarrow$ Sellers' merger is harmful for $\beta \in[0,1 / 4)$ and beneficial for $\beta \in(1 / 4,1]$. l.e. $\bar{\beta}_{J}=1 / 4$.
- In Example 2, since every buyer buys a unit from every seller, the allocation is not separable.
$\rightarrow$ Sellers' merger is beneficial for $\beta \in(0,1]$ and neutral for $\beta=0$.


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## Substitutability \& the Separability

Consider the $2 \times 2$ market ( $i=A, B, \& j=C, D$ ) where

- linear inverse demand $p_{j}^{i}=a_{i j}-b_{i} x_{j}^{i}-d_{i} x_{-j}^{i}$,
- $d_{i} \in\left[-b_{i}, b_{i}\right]$ is $i$ 's degree of substitutablility,
- $d_{i}=b_{i ;}$ products are perfect substitutes.
- $d_{i}=-b_{i} ;$ products are perfect complements,
- linear cost $c_{j}\left(x_{j}^{A}+x_{j}^{B}\right)\left(\right.$ marginal cost is $\left.c_{j}\right)$,
- $a_{A C} \geq a_{B C} \& a_{B D} \geq a_{A D}(A(B)$ prefers $C(D)$ more than $B(A))$.

Then, the core allocation is separable if and only if

- $d_{A} \geq b_{A} \frac{a_{A D}-c_{D}}{a_{A C}-c_{C}} \& d_{B} \geq b_{B} \frac{a_{B C}-c_{C}}{a_{B D}-c_{D}}$.
$\rightarrow$ As $d_{i} \nearrow$, the core allocation tends to be separable.

Let $s \geq 3$. A partial merger of two sellers is also harmful,

- if the core allocation is separable \& power $q$ is weak.
e.g. There are buyers $A, B$ \& Sellers $C, D, E$.
- If the allocation is separated as $\{(A, C, D),(B, E)\}$,
- the merger of $(C, E)$ is harmful to $C, E$.
- the coalitional deviation (., $C, D$ ) is possible before the merger, but impossible after the merger (only (., C, E) and (., C, D, E) are possible).


## A Merger Game

Suppose that buyers (Bs) \& sellers (Ss)

- choose merge or not merge, simultaneously.

Then, $\mathrm{Ss}^{\prime}$ choice is merge even if $q$ is low in an eqm.

- If $q$ is low, Bs have incentive to merge because Bs are proposers and extract full surplus w.p. $1-q$.
- Ss rationally infer that Bs' choice is merge.
- Ss' best response to Bs' merge is merge because
w.p. $1-q$, sellers' full surplus is extracted by the merged Bs irrelevant to Ss' choice (the merger of responders is neutral),
w.p. q, Ss can extract full buyers' surplus if merge, but cannot if not merge.


## Implication for Merger Review

Suppose that the core allocation is separable (high substitutability).

- If sellers' merger is applied even when $q$ is low,
- this merger is supposed to improve the welfare (e.g. by cost reduction).
$\because$ Otherwise, the merger is harmful.
Thus, the primal factor of merger review is vertical bargaining power.
cf. In usual, the primal factor is the market share (concentration).
- The merger of powerful sellers must be reviewed carefully.
- By contrast, the regulator can approve the merger of powerless sellers without the costly review.


## Outline

## Introduction

## Examples

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## Results

## Discussion

Summary

## Conclusion

We show that even the merger of sellers to monopoly

- is harmful if vertical power $\beta$ is low.
$\because$ a low-cost seller loses the power to switch.
The crucial assumption is
- the merger does not raise the power $\beta$ significantly.
$\because$ The merger to monopoly may increase the power of merging side. Our contribution might be
- a decomposition of benefit/harm of the merger to monopoly into
(i) the vertical bargaining effect (negative), and
(ii) the horizontal imperfect competition effect (positive).
- When $\beta \approx 0$, (ii) is almost zero, but (i) is negative $\rightarrow$ harmful.
- When $\beta \approx 1$, (i) is almost zero and (ii) is positive $\rightarrow$ beneficial.

