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Ratifiable Collusion and Bidding Systems in Procurement

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Ratifiable Collusion and Bidding Systems in Procurement

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Abstract

This study explores stability in efficient collusion in government procurement auctions. In first- and second-price auctions with independent private values, we look at the possibility of vetoing collusion mechanisms and the learning of the other bidders after vetoing. The collusions in first-price auctions in simple case and second-price auctions are stable against the competition after a potential veto to take part in bid-rigging.

Keywords: auctions, bid rigging, collusion, procurement, ratifiability.

JEL Classifications: C72, D44, D82, L44, H57.

1 Introduction

Bid-rigging in procurement auctions is serious problem in many countries with regard to maintaining competition . In Japan, cases of collusion in procurement auctions represent the majority of violations in the Antimonopoly Act. While many authors study collusion in auctions, we need undertake further investigation to gain useful insights into methods for to detecting and preventing bid-rigging. This study considers the effects of two auction formats on collusive behavior. We show that collusions in both these auctions are robust even if bidders have an option to deviate from bid-rigging and compete in auctions after vetoing. In the terms of auction theory, our result also adds knowledge of the similarity between these auctions.

Graham and Marshall (1987) extensively analyze collusion in second-price auctions for selling an item, and Mailath and Zemsky (1991) extend their model to asymmetric bidders. McAfee and McMillan (1992) analyze an efficient collusion in first-price auctions. Robinson (1985) compares possibilities to collude in first-price and second-price auctions. Holt (1980) shows that the choice between first- and second-price auctions gives significant impact on the expected cost of procurement when bidders are risk averse. There are a lot of empirical literature about collusion in procurement auctions. For example, Bajari and Ye (2003) study a method to identify bid rigging and find asymmetries in collusive behaviors as well as competitive bidder's costs due to location or capacity constraint.

In these theoretical models each bidder takes part in bid-rigging if incentive and individually rational conditions for collusion are satisfied. In reality, we observe breakdowns of collusion and thereafter competitive behavior.[1] A bidder has an opportunity to veto bid-rigging, and other riggers may be able to learn the cost information of the vetoer from his veto.[2] Cramton

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[1]While there are many triggers of breaking cartels, Levenstein and Suslow (2006) argue in their collected data that the mean duration of a cartel is 17.2 years and the median is 8 years.

[2]Hinloopen and Soetevent (2005) carried out experimental analysis on a leniency program in a Bertrand competition setting. Winning defectors among the subjects fre-

and Palfrey (1995) present an elaborate concept of *ratifiability* in a mechanism design setting to analyze such situations. If a proposed mechanism is robust against some status quo mechanism after a veto, the mechanism is called *ratifiable*. They investigate the ratifiability of the monopoly outcome mechanism that is analyzed in Cramton and Palfrey (1990) against Cournot competition in two firms and linear demand. Since a low-cost firm prefers Cournot competition to collusion, collusion for the monopoly outcome is not ratifiable against the status quo. In terms of learning type after a veto, collusion in the quantity competition is not stable.

Tan and Yilankaya (2007) consider the ratifiability problem of bidder collusion in second-price auctions for selling an object with participation cost. The collusion in the auctions they consider is not ratified against the post second-price competitive auctions. However, we show that collusion in second-price auctions for buying an object with no participation cost is ratifiable against the post auctions.

In this study we investigate the possibilities of colluding in first-price and second-price procurement auctions. A government procures an item from potential sellers in these auctions with no participation cost. The former auction corresponds to sealed low-bid auctions that governments commonly adopt. When values are private, the latter is strategically equivalent to *open descending auctions*. In our assumptions, the collusion in first-price auctions with uniform distribution and two bidders and general second-price auction are ratifiable. Bid-rigging in procurement auctions is relatively stable. When a cartel is strong, even if a procurer changes a sealed bid auction to an open auction, we expect that the cartel remains effective against the reform. Since the collusion in the Cournot competition is ratifiable (Cramton and Palfrey (1995)), auction environments are more vulnerable against collusion than the quantity competition. To detect and prevent bid-riggings, procurers in many countries reform bidding systems and competition authorities use some tools, e.g., leniency programs,

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quently apply leniency to protect their profits. The defection and veto on collusion may bring some information about the exerciser.

etc. Our conclusion partially justifies these efforts to weaken incentives to commit bid-riggings. Since the participation cost restores competition (Tan and Yilankaya (2007)), their conclusion and our analysis imply that some kind of a burden for bidders before auctioning may be a key to preventing collusion.

Section 2 presents basic models and preliminary results compared with the standard auction literature. In Section 3 we provide an efficient collusion mechanism. Section 4 introduces ratifiability. We obtain ratifiability results in first-price auctions in Section 5 and second price-auctions in Section 6. Section 7 concludes.

2 Model and Noncooperative Auctions

A government seeks to buy an item from N potential sellers. Buyer's reserve price is r. When the item is a public work, we may consider the reserve price to be an engineering cost estimate for the project. If the lowest bid is above r in each auction, the government does not buy it. In each auction, the lowest price bidder becomes a winner if the price is below or equal to the reserve price. When a tie occurs, to avoid the non-existence problem for an asymmetric case in the first price auction, we adopt *status quo tie-breaking rule*.[3] In this case the object is surely given to one of the bidders who quote the lowest price. In a first-price auction, a winner receives his quoted price, but, in a second-price auction he receives a price equal to the second lowest price if the price is below or equal to the reserve price. Otherwise, he receives the reserve price.

The sellers' reservation utility is normalized to 0. They are risk neutral. Our model is a symmetric independent private value one. Each bidder i has cost c_i that is distributed according to a common distribution function F on $[\underline{c}, \overline{c}]$ with density function f, which is strictly positive on the domain. Each bidder's cost is his private information. He infers the others costs from the

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[3]See Tanno (2008) for more detail. In symmetric environment or second-price auctions, the tie-breaking rule also works for equilibrium.

prior probability.

Here we consider a competitive first-price auction as a benchmark. We assume that all bidders use common nondecreasing strategy $p(c) < r$ for a cost c. Let $\pi^1(c|r)$ be a profit for a representative supplier with cost c in the first-price auction. The standard argument implies

$$\pi^1(c|r) = \max_p (p - c)(1 - F(p^{-1}(p)))^{N-1}$$

where $p^{-1}(\cdot)$ is an inverse function for $p(\cdot)$. By the envelope theorem and $p^{-1}(p) = c$,

$$\frac{d\pi^1(c|r)}{dc} = -(1 - F(c))^{N-1}.$$

By integrating the above profit, we obtain the bidder's equilibrium profit $\pi^{1*}(c|r)$:

$$\pi^{1*}(c|r) = \begin{cases} 0 & \text{(if } c > r) \\ \int_c^r (1 - F(x))^{N-1} dx & \text{(if } c \leq r) \end{cases} \tag{1}$$

Substituting $\pi^{1*}(c|r)$ in the objective function, the bidding function given the reserve price r is

$$p^{1*}(c|r) = \begin{cases} No & \text{(if } c > r) \\ c + \int_c^r \left(\frac{1-F(x)}{1-F(c)}\right)^{N-1} dx & \text{(if } c \leq r) \end{cases}$$

where No means not participating.

Next, we consider a competitive second-price auction. By the standard argument, submitting true cost is a weakly dominant strategy for a bidder whose cost is not above the reserve price. Then, the equilibrium bid $p^{2*}(c|r)$ is given by

$$p^{2*}(c|r) = \begin{cases} No & \text{(if } c > r) \\ c & \text{(if } c \leq r). \end{cases}$$

Let $\pi^{2*}(c|r)$ be a profit for a representative supplier with cost c in the second-price auction. Since the density of the lowest order statistic among

$N - 1$ is $(N - 1)(1 - F(x))^{N-2}f(x)$, a profit for a bidder with cost $c \leq r$ is given by

$$\pi^{2*}(c|r) = (r - c)(1 - F(r))^{N-1} + \int_c^r (x - c)(N - 1)(1 - F(x))^{N-2}f(x)dx.$$

(2)

Note that the first term represents the profit in the case where all the rivals bid r or do not take part in the auction. Of course, the situation in this section satisfies the conditions under which the revenue equivalence theorem holds. A profit $\pi^{1*}(c|r)$ for cost c supplier in the first-price auction is equal to (2).

We briefly study welfare analysis in the auctions.[4] The item that the government purchases has a social benefit S. However, the procurement entails some distortionary cost $\lambda > 0$ to raise a fund to buy it through taxation. When the payment made by the government to a bidder is p, the consumer surplus is $S - (1 + \lambda)p$. Adding expected profits for all firms, the welfare W^* can be easily computed:

$$W^* = N \int_{\underline{c}}^r \left(S - (1 + \lambda)c - \lambda\frac{F(c)}{f(c)} \right)(1 - F(c))^{N-1}f(c)dc.$$

Note that by the revenue equivalence theorem the welfare is identical for the two auctions.

We can choose the optimal reserve price in the way that Myerson (1981) analyzes optimal auctions with a regularity condition. Let

$$J(c) = (1 + \lambda)c + \lambda\frac{F(c)}{f(c)}$$

be the *virtual cost with shadow cost*. We assume that the virtual cost is monotonically increasing in c and $S > J(\underline{c})$. Just as Bulow and Roberts (1989) recognize a virtual valuation in auctions for a seller as an expected marginal revenue, we think $J(c)$ as the expected marginal cost generated if a bidder of type c supplies the item. Since the government faces a zero-one problem about whether to procure one item or not under uncertainty, we

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[4]See Miura (2003) for further details on the following analysis in this section.

can also regard the social benefit S as a "marginal social benefit." Actually, differentiating W^* leads to

$$W^{*'} = N(S - J(r))(1 - F(r))^{N-1}f(r).$$

If $S < J(\bar{c})$, to maximize the social welfare, the government attempts to set marginal benefit equal to marginal cost as the way to select reserve price r^* such that $S = J(r^*)$. Namely,

$$S = (1 + \lambda)r^* + \lambda\frac{F(r^*)}{f(r^*)}. \tag{3}$$

If the social benefit is sufficiently large such that $S \geq J(\bar{c})$, the optimal reserve price is $r = \bar{c}$. In this case it is a corner solution and the government invites all types of firms.

3 Collusion

We consider an all-inclusive collusion with transfer. All bidders collude and coordinate their behavior through the direct mechanism as follows. Before an auction, the cartel members report their costs to the mechanism. If all bidders' reported costs are above or equal to the reserve price, the cartel does not bid in the auction. If at least one bidder's reported cost is lower than the reserve price, the bidder making the lowest report quotes the reserve price, the others bid a price higher than the reserve price, and the lowest report bidder sells the item by the price r and transfers his revenue equally to each losing bidder. If the collusion scheme is feasible, the outcome in the first-price auction is the same as that in the second-price auction.

McAfee and McMillan (1992) show that there exists such a mechanism that is incentive-compatible, ex post efficient, and ex post budget balanced. The collusive payoff π^m for a firm that has cost c and reports $\hat{c}$ is given by

$$\pi^m(\hat{c}, c|r) = (r - c - T(\hat{c}))(1 - F(\hat{c}))^{N-1} +$$
$$\int_{\underline{c}}^{\hat{c}} \frac{T(x)}{N-1} \cdot (N-1)(1 - F(x))^{N-2}f(x)dx \tag{4}$$

where $T(\hat{c})$ is the total transfer to the other losing bidders and $(N-1)(1-F(x))^{N-2}f(x)$ is the density of the lowest cost among $N-1$ bidders. By the revelation principle without loss of generality we can restrict attention to truthful report. By McAfee and McMillan (1992), we easily obtain an incentive compatible transfer.[5]

Lemma 3.1 (McAfee and McMillan (1992)) *In a symmetric, ex post efficient, ex post budget balance, and an incentive compatible cartel, the total transfer from a winning cost-c bidder to losing bidders is given by*

$$T(c) = \int_c^r (r-x)\frac{(N-1)(1-F(x))^{N-1}f(x)}{(1-F(c))^N}dx \qquad \text{for } c \in [\underline{c}, r).$$

Proof: By Guesnerie and Laffont (1984), to show incentive compatibility we check two conditions:

$$\frac{\partial^2 \pi^m(\hat{c}, c|r)}{\partial \hat{c} \partial c}\bigg|_{\hat{c}=c} > 0 \quad \text{and} \quad \frac{\partial \pi^m(\hat{c}, c|r)}{\partial \hat{c}}\bigg|_{\hat{c}=c} = 0.$$

By (4),

$$\frac{\partial \pi^m}{\partial \hat{c}} = -T'(\hat{c})(1-F(\hat{c}))^{N-1} - (N-1)(r-c-T(\hat{c}))(1-F(\hat{c}))^{N-2}f(\hat{c}) +$$
$$T(\hat{c})(1-F(\hat{c}))^{N-2}f(\hat{c}).$$

We easily get $\partial^2 \pi^m(\hat{c}, c|r)/\partial \hat{c} \partial c > 0$. Furthermore,

$$\frac{\partial \pi^m}{\partial \hat{c}}\bigg|_{\hat{c}=c} = -T'(c)(1-F(c))^{N-1} + T(c)N(1-F(c))^{N-2}f(c) -$$
$$(N-1)(r-c)(1-F(c))^{N-2}f(c)$$
$$= (N-1)(r-c)(1-F(c))^{N-2}f(c)$$
$$-N(N-1)(1-F(c))^{-2}f(c)\int_c^r (r-x)(1-F(x))^{N-1}f(x)dx$$
$$+N(N-1)(1-F(c))^{-2}f(c)\int_c^r (r-x)(1-F(x))^{N-1}f(x)dx$$
$$-(N-1)(r-c)(1-F(c))^{N-2}f(c) = 0.$$

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[5]We note that the transfer T in Lemma 3.1 is symmetrical with the one in Theorem 3 in McAfee and McMillan (1992).

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The other properties are obvious. ∎

When $\hat{c} = c$, we simply write $\pi^m(c|r)$. We characterize collusive payoffs.

Lemma 3.2 *In a symmetric, ex post efficient, ex post budget balance, and an incentive compatible cartel, the expected payoff for a cost-c bidder is given by*

$$\pi^m(c|r) = \begin{cases} \pi^m(r|r) & \text{(if } c > r) \\ \pi^m(r|r) + \int_c^r (1 - F(x))^{N-1} dx & \text{(if } c \leq r) \end{cases} \tag{5}$$

where

$$\pi^m(r|r) = \int_{\underline{c}}^r (r - x)(N-1)(1 - F(x))^{N-2} F(x) f(x) dx.$$

The proof is contained in Appendix A. We note that the second term of $\pi^m(c|r)$ for $c \leq r$ is equal to the noncooperative payoff (1). This stems from incentive compatibility.[6] Since the mechanism must give an incentive to the most inefficient firm to tell the true cost, cost-r firm is given by a transfer and his profit is positive.[7] Since the density of the second-lowest order statistic among N is $N(N-1)(1-F(x))^{N-2}F(x)f(x)$, the fixed term $\pi^m(r|r)$ is equal to the expected value of $1/N$ times the profit that the second most profitable firm would earn when he quotes price r on the item. Therefore, the highest-cost firm must receive an equal share of the expected profit when the second-lowest-cost firm supplies the item. The winning bidder's total rent for the auctions is the difference between the winner's (lowest) cost and the second-lowest cost. The winner takes the rent. The

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[6]By the theorem in Guesnerie and Laffont (1984), we need to have $d\pi^m(c|r)/dc = -(1 - F(c))^{N-1}$. The derivative is the same as that of first-price auction in the envelope theorem.

[7]The result is similar to Lemma 4 in Cramton and Palfrey (1990) for the monopoly mechanism and a formula in the proof of Theorem 4 in McAfee and McMillan (1992) for a seller's auction.

remaining surplus, which is the difference between the reserve price and the second lowest cost, is split equally among the cartel members (including the winner). This collusion mechanism can be implemented by a "preauction knockout," which Graham and Marshall (1987) analyze.

Obviously, $\pi^m(r|r) > 0$ in Lemma 3.2 and $\pi^{k*}(r|r) = 0$ for noncooperative profit in each k. The collusive profits are higher than the noncooperative profits for each cost.

Lemma 3.3 $\pi^m(c|r) > \pi^{k*}(c|r)$ *for* $k = 1, 2$ *and all* $c \in [\underline{c}, \overline{c}]$.

We assume that the government knows the existence of the cartel, but cannot crack down on it. Welfare W^m under the collusive scheme is reduced to

$$W^m = N \int_{\underline{c}}^{r} \left(S - \lambda r - c \right) (1 - F(c))^{N-1} f(c) dc.$$

We assume the concavity of W^m. If the government optimally responds to the cartel, the optimal anticartel reserve price r^m is given by

$$S = (1 + \lambda) r^m + \lambda \frac{1 - (1 - F(r^m))^N}{N(1 - F(r^m))^{N-1} f(r^m)}.$$

The formula is similar to (3). We can deduce the implication from the latter. Remember that in the case of competitive bidding the problem is to find a "marginal supplier" at an appropriate reserve price. Since under the collusion all N bidders potentially quote a reserve price, the probability that some bidder sells the item at price r^m is $1 - (1 - F(r^m))^N$ and its density is $N(1 - F(r^m))^{N-1} f(r^m)$. The ratio of the probability to the density is the second term in the right-hand side. This implies that the anticartel reserve price decreases with the number of cartel members and is lower than the optimal reserve price under the same shadow cost λ.[8]

4   Ratifiability

In the previous section we see that collusive profits are higher than competitive bidding profits. However, we sometimes observe a voluntary deviation

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[8]See Graham and Marshall (1987) and McAfee and McMillan (1992) for more details.

from bid-riggings. While deviators may be threatened by a competition authority, there may be a problem with the stability of cartels.

Since a winning deviator does not need to pay transfers to losers in bid-riggings, a low-cost firm with a high possibility of winning may prefer competitive auctions to bid-rigging. Although the collusion scheme is incentive compatible, a competition after the deviation could possibily change bidders' beliefs. The others can infer his cost by observing his deviation. They may anticipate that he has low cost. Thus, they may leave the auction. If he successfully makes them believe him to be low cost, he may gain a monopoly profit. In the following sections we introduce the concept of *ratifiability* to analyze the possibility of the above situation. We examine whether a difference in auction formats can influence collusion in the presence of the opportunity not to take part in collusion and to learn from deviation.

Cramton and Palfrey (1995) formulate the following two stage game to present ratifiability in a mechanism design environment. After firms know their own costs, at the first stage each firm votes for the proposed mechanism (bid-rigging). At the second stage the mechanism or the status quo game (first- or second-price auctions) is played. If they are unanimously for the mechanism, it is played. Otherwise, the status quo is played with the knowledge that some player vetoes the mechanism. Therefore, when firm i decides whether to veto the mechanism, he must consider how the others' beliefs about i might change as a result of the veto. There are many possibilities about beliefs and refinement concepts.[9] One of possibilities for beliefs is passive belief. When we adopt passive belief as off-equilibrium belief and in equilibrium the mechanism is preferable to the status quo game for any type for any player, unanimous ratification of the mechanism is a

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[9]See Cramton and Palfrey (1995) for more detailed discussion. To avoid the refinement argument, Caillaud and Jehiel (1998) consider the payoff after vetoing as the smallest interim payoff where the minimum is taken over all possible beliefs held by the other buyers of his type.

sequential equilibrium in the two stage game.[10] In this case the mechanism is called *individually rational relative to the status quo game*. By Lemma 3.3 the bid-rigging mechanism is individually rational relative to competitive auctions.

However, this cannot formulate the learning from a veto. We now introduce more subtle beliefs and a refinement concept. If firm i vetoes the mechanism, the others believe that a type of i is in $V_i \subset [\underline{c}, \overline{c}]$ and each type in V_i vetoes with positive probability. We call such a set a V_i *veto set* for i. The veto belief F_v is induced by the prior F and the veto probabilities. Let $\pi_i^{k*}(c|r)$ be cost-c bidder i's payoff in post competitive k-auctions ($k = 1$ for first-price and $k = 2$ for second-price auctions) and $U_i^k(c|r) = \pi^m(c|r) - \pi_i^{k*}(c|r)$.[11]

Definition 1 *Fix $k = 1$ or 2. The veto set V_i is credible about i relative to the mechanism and the status quo game k if there exist an equilibrium in the status quo game, veto probabilities, and posterior beliefs such that*

(1) there exists a positive veto probability in some type,

(2) $c \in V_i$ for $U_i^k(c|r) < 0$,

(3) $c \notin V_i$ for $U_i^k(c|r) > 0$,

(4) the posterior beliefs of the others about i are updated by his prior belief and veto probabilities using Bayes' rule.

A type in a credible veto set has an incentive to veto the mechanism. By his veto the others believe that his type is in V_i and he benefits from it. We note that we put no restriction on types that are indifferent between vetoing and not. The ratifiability concept depends on the indifference.

Definition 2 *Fix $k = 1$ or 2. The incentive compatible mechanism is ratifiable against the status quo game k if the mechanism is individually rational relative to the status quo game and for all i either*

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[10]See Proposition 1 in Cramton and Palfrey (1995).

[11]Payoffs depend on a post-veto strategy and belief. However, by symmetry and simplicity, we suppress strategy and belief in the expression.

(1) there does not exist a credible veto set for i, or

(2) there exists a credible veto set such that $U_i^k(c|r) = 0$ for all $c \in V_i$ under a corresponding equilibrium and beliefs.

Cramton and Palfrey (1995) show that a monopoly outcome mechanism against Cournot competition in two firms and a linear demand case is not ratifiable. Tan and Yilankaya (2007) present the non-ratifiability result of bidder collusion in second-price auctions with participation cost for selling an object. We will show that the cartel mechanism against competitive first- or second-price auctions is ratifiable. The result implies that the bid-rigging is relatively stable.

5 Ratifiability in First-Price Auctions

To analyze ratifiability we must be cautious of asymmetry induced by belief change. Analysis of asymmetry in first-price auctions is especially difficult task and was not done until the late 1990s. Maskin and Riley (2000a) compares revenues between first- and second-price auctions. Maskin and Riley (2000b) treats the monotonicity of an equilibrium bid. Lebrun (1999) shows the existence of equilibrium. Moreover, there are few explicit solutions. Griesmer, Levitan, and Shubik (1967), Kaplan and Zamir (2007), and Tanno (2008) give explicit solutions to an auction with two bidders and uniform distribution.

In this case, we cannot use the standard technique to induce an equilibrium in symmetric auctions. We must directly solve differential equations with boundary conditions. Since it is difficult to obtain a general solution in asymmetric first-price auctions, we prove a ratifiable result in the first-price auction with two suppliers and uniform distribution. In the next section we examine differential equations and boundary conditions to solve the ratifiability problem.

5.1 General First-Order Condition

For the moment, we consider asymmetric bidders with a cost c_i that is distributed according to a distribution function F_i on a support $[\underline{c}_i, \overline{c}_i]$. Using the inverse bid function $c_i = c_i(p_i)$, general first order conditions are given by

$$-(p_i - c_i) \sum_{j \neq i} F'_j c'_j \Pi_{k \neq i,j}(1 - F_k) + \Pi_{j \neq i}(1 - F_j) = 0 \qquad \text{for } i = 1, \ldots, N$$

where F'_j, c'_j, and F_k are evaluated at p_i, $c_j(p_i)$, and $c_k(p_i)$. Following Lebrun (1999), Maskin and Riley (2000a), and Bajari and Ye (2003), equilibrium inverse bid functions are characterized by solutions to these differential equations

$$c'_i = \frac{1 - F_i}{(N-1)F'_i} \left(\sum_{j \neq i} \frac{1}{p - c_j} - \frac{N-2}{p - c_i} \right) \qquad \text{for } i = 1, \ldots, N$$

where c_i, c_j, F_i, and F'_i are evaluated at p and $c_i(p)$ over $(\underline{p}, \overline{p})$ satisfying some boundary conditions.

Since the belief for the vetoer is updated to some distribution function F_v with a veto set $V_v = [l, h]$, which is a subset of the original support, and that of the other sellers is unchanged,[12] the differential equations can be reduced to

$$c'_v = \frac{1 - F_v}{(N-1)F'_v} \left(\sum_{j \neq v} \frac{1}{p - c_j} - \frac{N-2}{p - c_v} \right), \tag{6}$$

$$c'_i = \frac{1 - F_i}{(N-1)F'_i} \left(\sum_{j \neq i} \frac{1}{p - c_j} - \frac{N-2}{p - c_i} \right) \qquad \text{for } i \neq v.$$

Analytic solutions for general distributions are rare in first-price auctions. Following Kaplan and Zamir (2007) and Tanno (2008), we derive a ratifiability result for a simplified auction in the case of two suppliers and uniform distribution.

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[12]By Propositions 1 and 3 in Maskin and Riley (2000b), the support of the distribution of winning bids is an interval and the c.d.f. of the winning bids is continuous on the interval. This implies that we may concentrate our attention on an interval for an updated belief.

5.2 Two bidders and Uniform Distribution Case

We consider two bidders case. Each prior cost is uniformly distributed on $[0, 1]$. In status quo game a cost for ratifier is distributed according to $F_r(c) = c$. We consider a candidate for veto set $V_v = [l, h]$ where $0 \leq l < h \leq 1$. The ratifier's belief about the vetoer is updated by Bayes' rule to $F_v(c) = (c - l)/(h - l)$. By (6), a system of differential equations are

$$(p - c_v)c_r'(p) = 1 - c_r(p) \quad \text{and} \quad (p - c_r)c_v'(p) = h - c_v(p). \quad (7)$$

There are two types of solutions. First, we consider a "linear bid" as a particular solution in (7). Second, we consider another nonlinear solution that Kaplan and Zamir (2007) extensively analyze. In each equilibrium we cannot find a credible veto set for any reserve price.

Note that if the vetoer's equilibrium bid is equal to a reserve price, such a high-cost vetoer has no incentive to take part in the auction. The cartel mechanism ensures a positive profit for a high-cost bidder. Therefore, we may focus on a veto set in which the reserve price is not effective. Lemma 3.2 gives us the collusive payoff:

$$\pi^m(c|r) = \pi^m(r|r) + \int_c^{\bar{c}} (1 - F(x))dx = \frac{1}{6}(r^3 - 3(r - c)^2 + 6(r - c)). \quad (8)$$

Of course, any cartel member likes a high reserve price:

$$\frac{\partial \pi^m(c|r)}{\partial r} = \frac{1}{2}((r - 1)^2 + 2c + 1) > 0 \quad \text{for all } c. \quad (9)$$

Since we show that there does not exist any credible veto set and each cost type in a credible veto set is better off in the status quo game than in the cartel mechanism, it is sufficient to consider the payoff difference at the lowest reserve price.

The linear solution for the above differential equations is

$$c_v(p) = 2p - \frac{2 + h}{3} \quad \text{and} \quad c_r(p) = 2p - \frac{1 + 2h}{3}.$$

Since this solution is a particular one, we need no boundary condition.[13] The price that the highest cost vetoer bids becomes the highest equiblirum

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[13]Note that when $h = 1$, the solution is equal to those for symmetric case.
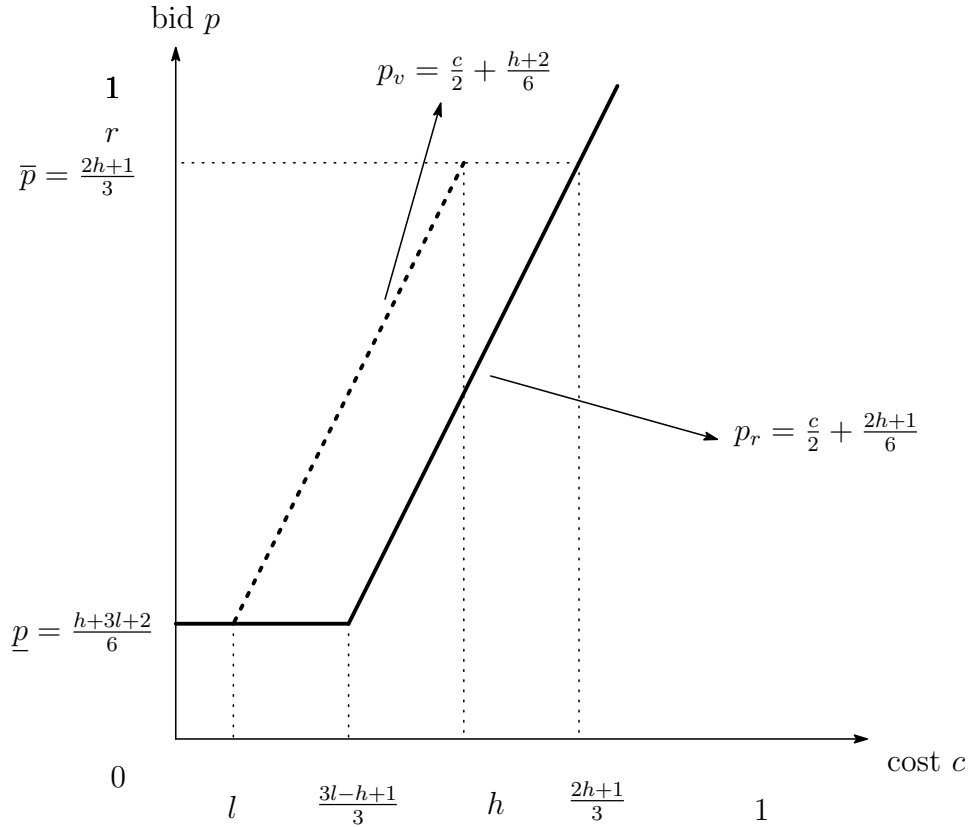
Figure 1: Two bidders and uniform distribution case in the first price auction

price $\overline{p}$. We assume $r \geq \overline{p}$ by the above reason. While any ratifier who has cost higher than the cost corresponding to this highest price has zero-probality of winning, in equilibrium this type of the ratifier still follows c_r. By the status quo tie-breaking rule in favor of vetoer a low-cost ratifier who can set a price below the lowest price $\underline{p}$ the vetoer bids quotes the price $\underline{p}$ and always beats any type of the vetoer except l-type vetoer.[14] By taking account of conditions for high and low ends, we obtain the proposition.

Proposition 5.1 *Assume that the status quo tie-breaking rule in favor of vetoer, $l \geq 0$, $h \leq 1$, and $r \geq \frac{2h+1}{3}$ in the first-price auctions with uniform*

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[14]See Tanno (2008) for the reason why the status quo tie-breaking rule restores equilibrium in asymmetric auctions.

distributions. The following strategies constitute equilibrium:

$$p_v = \frac{c}{2} + \frac{h+2}{6} \qquad \text{for } c \in [l, h],$$

$$p_r = \begin{cases} \frac{c}{2} + \frac{2h+1}{6} & \text{for } c > \frac{3l-h+1}{3} \\ \underline{p} = \frac{h+3l+2}{6} & \text{for } c \le \frac{3l-h+1}{3}. \end{cases}$$

See Tanno (2008) for the formal proof of the equilibrium bids. Prices and costs for each bid function are depicted in Figure 1.[15] Kaplan and Zamir (2007) also consider this type of solution. However, Proposition 5.1 shows the existence of a linear solution for a wider range of parameters than that in Kaplan and Zamir (2007).

The vetoer's profit derives from Proposition 5.1:

$$\pi_v^{1*}(c|r) = \frac{(h+2-3c)^2}{18} \qquad \text{for } c \in [l, h].$$

Next, we compare a collusive payoff with the status quo game payoff. According to (9), it suffices to consider the possible minimum reserve price. We substitute $\underline{r}(h) = (2h+1)/3$ in the supposition of Proposition 5.1 into (8) and gets the payoff difference:

$$U_v^1(c|\underline{r}(h)) = \pi^m(c|\underline{r}(h)) - \pi_v^{1*}(c|\underline{r}(h))$$
$$= \frac{1}{162}(8h^3 - 33h^2 + 6(7+27c)h - 162c^2 + 10) \qquad \text{for } c \in [l, h].$$

We differentiate the above expression with respect to h:

$$\frac{\partial U_v^1}{\partial h}(c|\underline{r}(h)) = \frac{1}{27}(4h^2 - 11h + 7 + 27c) = \frac{1}{27}\left(4\left(h - \frac{11}{8}\right)^2 + 27c - \frac{9}{16}\right).$$

We see that $\frac{\partial U_v^1}{\partial h}(c|\underline{r}(h))$ decreases in h for a feasible interval. The value at the maximum upper end $h = 1$ is

$$\frac{\partial U_v^1}{\partial h}(c|\underline{r}(1)) = c \ge 0 \qquad \text{for all } c.$$

---

[15]We observe that when each bidder takes part in auctions and their behavior is described by the differential equations, the ratifier is more aggressive than the vetoer since $p_v(c) = c/2 + (h+2)/6 > p_r(c) = c/2 + (2h+1)6$. The observation confirms proposition 3.3 in Maskin and Riley (2000a).

Therefore, we obtain

$$\frac{\partial U_v^1}{\partial h}(c|\underline{r}(h)) \geq 0 \qquad \text{for all } c \leq h \text{ for all } h \in (0, 1].$$

Then ,we can calculate the minimum of the difference of payoffs by taking limit of h and c to 0.

$$\lim_{h,c\to 0} U_v^1(c|\underline{r}(h)) = \frac{5}{81} > 0.$$

Therefore, we conclude that for any veto set V_v and reserve price r,

$$U_v^1(c|r) > 0 \qquad \text{for any } c \in V_v.$$

In this type of equilibrium, there is no credible veto set.

By following Kaplan and Zamir (2007) we analyze the payoff difference at another equilibrium. This solution is not linear and generalizes the solution that Griesmer, Levitan, and Shubik (1967) consider. The boundary condition is a little different from that for the previous linear solution.[16]

Proposition 5.2 (Kaplan and Zamir (2007)) *If the probability of winning is zero, each bidder bids his own cost. Assume that $l \geq 0$, $h \leq 1$, and $r \geq \frac{2h+1}{3}$ in the first-price auctions with uniform distributions. The equilibrium inverse bid function for vetoer and ratifier is given by*

$$c_v(p) = h + \frac{(1-h)^2}{(1+h-2p)K_v e^{\frac{1-h}{1+h-2p}} + 4(1-p)},$$

$$c_r(p) = 1 + \frac{(1-h)^2}{(1+h-2p)K_r e^{-\frac{1-h}{1+h-2p}} + 4(h-p)},$$

where

$$K_v = -\frac{\frac{(1-h)^2}{h-l} + 4(1-\underline{p})}{2(\overline{p} - \underline{p})} e^{\frac{1-h}{2(\overline{p}-\underline{p})}}, \quad K_r = -\frac{(1-h)^2 + 4(h-\underline{p})}{2(\overline{p} - \underline{p})} e^{\frac{1-h}{2(\overline{p}-\underline{p})}},$$

$$\overline{p} = \frac{1+h}{2}, \quad and \quad \underline{p} = \frac{(1+h)^2}{4(1+h-l)}. \tag{10}$$

The costs for boundaries of the bids are $c_v(\overline{p}) = h$, $c_v(\underline{p}) = l$, $c_r(\underline{p}) = 0$, and $c_r(\overline{p}) = \overline{p}$.

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[16]See Kaplan and Zamir (2007) for the boundary condition for more detail.

See Proposition 1 in Kaplan and Zamir (2007) for the proof. While they analyze a first-price auction for selling an item, we can derive solution of an auction for buying an item in a similar way. While this equilibrium bid seems complicated, we can prove non-existence of a creadible veto set in this equilibrium to focus on the payoff at the lowest vetoer's cost. It is enough to show no existence of higher payoff in the competitive auction at the lowest cost for any veto set. See Appendix B for detailed proof.

By the investigation of the payoff difference in two equilibria, we conclude that there does not exist an equilibrium and posterior belief in the status quo game in which the vetoer is better off in the status quo game than in the collusion. We prove the following proposition.

Proposition 5.3 *In first-price auctions with two bidders and uniform distribution, the collusion mechanism is ratifiable against the status quo of competitive auctions.*

6 Ratifiability in Second-Price Auctions

In this section, we show that the collusion mechanism in a second-price auction is ratifiable. Compared with a first-price auction, we treat the general setting in second-price auctions. We consider the cases for N sellers and general support. When a bidder v vetoes, other bidders update their beliefs about the vetoer in the following way: the support of belief for the vetoer is $V = [l, h] \subset [\underline{c}, \bar{c}]$. The updated belief is $F_v(c) = (F(c) - F(l))/(F(h) - F(l))$. As we analyze the first-price auction, a bidder whose cost is above a given reserve price does not veto the collusion mechanism. We assume that the upper end is below or equal to the reserve price.

In the post-veto auction, a candidate for equilibrium is a weakly dominant strategy as is usual in second price auctions. In the case where ratifier i's cost is over h, the ratifier surely believes that he will lose to any vetoer and cannot gain from participation. Thus, choosing No is a weakly dominant strategy for this ratifier in this case. Using such a cut-off strategy, the
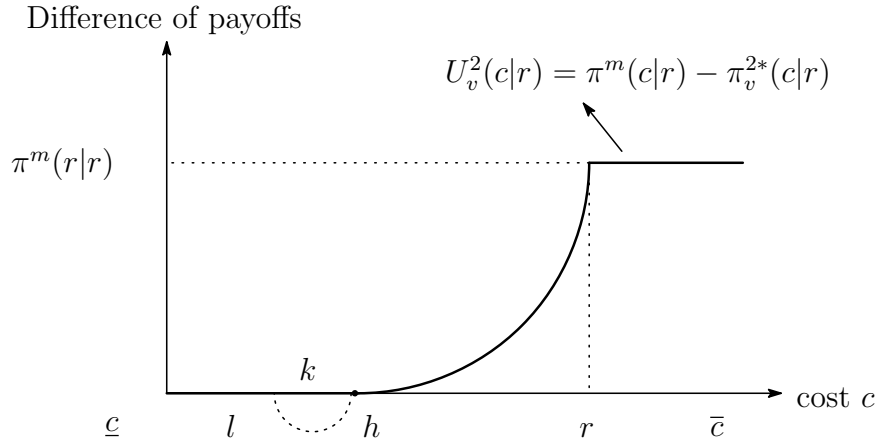
Figure 2: Payoff difference between collusion and post-veto auction

equilibrium $p^{2*} = (p_v^{2*}, p_i^{2*})$ in the post-veto auction is:

$$p_v^{2*}(c) = c \text{ for } c \in [l, h],$$

$$p_i^{2*}(c) = \begin{cases} No & \text{if } c \geq h \\ c & \text{if } c < h \end{cases} \text{ for } i \neq v.$$

Even if a ratifier's cost is below l, he certainly wins the auction by bidding his true cost.[17]

Proposition 6.1 *In the second price auctions the collusion mechanism is ratifiable against the status quo of competitive second price auctions.*

See Appendix C for the proof. The difference U_v^2 of the payoffs corresponds to the bold line in Figure 2. Even if the vetoer makes the ratifiers stay out, he only gets a payoff equal to the bid-rigging payoff in this case. Since the vetoer does not strictly gain by vetoing in any case, the candidate veto set is not credible. Since the equilibrium p^* is a weakly dominant strategy, there is no credible belief and no strategy pertaining to it. Our result is in contrast to the 'no ratifiability' result in Tan and Yilankaya (2007). The difference stems from the participation cost. Tan and Yilankaya (2007)

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[17]In the model of Tan and Yilankaya (2007) which consider a participation cost, the threshold of entry is different from the highest cost for the vetoer.

shows that if after realization of cost, a participation for bidding entails a positive entry cost for bidders, the collusion mechanism in the second-price auctions is not ratifiable against the status quo competitive auctions.

We explain the intuition behind the logic of the non-ratifiability result. A positive entry cost induces ratifiers not to take part in the auction for some costs. Let k be a threshold cost for entry. Since revenue for a winning cost-h ratifier could be at most h and his profit with participation cost would be not positive, k is strictly lower than h. Between h and k, without ratifiers, the vetoer gets more profit in competitive auction. If we choose a veto set in this area, we can show that such a veto set is credible. So, the collusion is not ratifiable. The correspondent payoff difference is depicted by the dotted line in Figure 2.

7   Concluding Remarks

We formulate an efficient collusion scheme in first- and second-price auctions for buying an object. In a procurement auction setting, to bring the possibility of vetoing collusion and the learning of the other bidders after vetoing, we investigate the stability of bid-rigging. We conclude that collusion in sealed low-bid auctions with uniform distribution and two bidders and general open descending auctions is stable against competition after a potential veto to take part in bid-rigging. The two leading bidding systems are not immune to collusion.

We compare our findings with related results. In a second-price auction, some participation cost hinders collusion. If we reform the bidding system to introduce some cost that influences not the production cost, but the entry decision, the new bidding system of open descending auctions is expected to reduce collusion. Collusion is easier in auctions than it is in market competition. Competition authority should be more careful with bid-rigging than price cartels. To detect and prevent collusions, procurers reform bidding systems and competition authorities use some tools, e.g., leniency programs, etc. Our conclusion partially justifies these efforts to

deter bid-riggings.

In future research we will investigate a bidding system in which a cartel is not ratifiable. To formulate better competition policy, it is important to investigate what conditions or institutional characteristics affect collusion.

A   Proof of Lemma 3.2

Proof: The collusive payoff for a firm with cost c is given by

$$\pi^m(c|r) = (r - c - T(c))(1 - F(c))^{N-1} + \int_{\underline{c}}^{c} T(x)(1 - F(x))^{N-2} f(x) dx$$

$$= (r - c)(1 - F(c))^{N-1}$$

$$- (N-1)(1 - F(c))^{-1} \int_{c}^{r} (r - x)(1 - F(x))^{N-1} f(x) dx$$

$$+ (N-1) \int_{\underline{c}}^{c} (1 - F(x))^{-2} f(x) \int_{x}^{r} (r - y)(1 - F(y))^{N-1} dy dx.$$

$$(11)$$

Note that by integral by parts,

$$\int_{c}^{r} (r - x)(1 - F(x))^{N-1} f(x) dx = \frac{1}{N} \left((r - c)(1 - F(c))^N - \int_{c}^{r} (1 - F(x))^N dx \right).$$

$$(12)$$

Applying (12) and a kind of (12) to the second and the third terms in (11), we get

$$\pi^m(c|r) = \frac{1}{N}(r - c)(1 - F(c))^{N-1} + \frac{N-1}{N}(1 - F(c))^{-1} \int_{c}^{r} (1 - F(x))^N dx$$

$$+ \frac{N-1}{N} \int_{\underline{c}}^{c} (r - x)(1 - F(x))^{N-2} f(x) dx$$

$$- \frac{N-1}{N} \int_{\underline{c}}^{c} (1 - F(x))^{-2} f(x) \int_{x}^{r} (r - y)(1 - F(y))^{N-1} dy dx. \quad (13)$$

We obtain that by changing the order of integration,

$$\int_{\underline{c}}^{c} \frac{\int_{x}^{r}(1-F(y))^{N}dy}{(1-F(x))^{2}}f(x)dx$$

$$=\int_{c}^{r}\int_{\underline{c}}^{c}\frac{(1-F(y))^{N}}{(1-F(x))^{2}}f(x)dxdy+\int_{\underline{c}}^{c}\int_{\underline{c}}^{y}\frac{(1-F(y))^{N}}{(1-F(x))^{2}}f(x)dxdy$$

$$=\int_{c}^{r}(1-F(y))^{N}((1-F(c))^{-1}-1)dy+\int_{\underline{c}}^{c}(1-F(y))^{N}((1-F(y))^{-1}-1)dy$$

$$=\int_{c}^{r}\frac{(1-F(x))^{N}}{1-F(c)}dx+\int_{\underline{c}}^{c}(1-F(x))^{N-1}dx-\int_{\underline{c}}^{r}(1-F(x))^{N}dx.$$

Using this equations and a kind of (12), (13) is equal to

$$\pi^{m}(c|r)=\frac{1}{N}(r-\underline{c})-\int_{\underline{c}}^{c}(1-F(x))^{N-1}dx+\frac{N-1}{N}\int_{\underline{c}}^{r}(1-F(x))^{N}dx$$

$$=\frac{1}{N}(r-\underline{c})+\frac{N-1}{N}\int_{\underline{c}}^{r}(1-F(x))^{N}dx-\int_{\underline{c}}^{r}(1-F(x))^{N-1}dx$$

$$+\int_{c}^{r}(1-F(x))^{N-1}dx.$$

Especially,

$$\pi^{m}(r|r)=\frac{1}{N}(r-\underline{c})+\frac{N-1}{N}\int_{\underline{c}}^{r}(1-F(x))^{N}dx-\int_{\underline{c}}^{r}(1-F(x))^{N-1}dx.$$

Next, we transform this fixed term. We note that

$$((r-x)(1-F(x))^{N-1}F(x))'=-(1-F(x))^{N-1}F(x) \tag{14}$$

$$-(r-x)(N-1)(1-F(x))^{N-2}F(x)f(x)+(r-x)(1-F(x))^{N-1}f(x).$$

Rearranging $\pi^{m}(r|r)$ gives

$$\pi^{m}(r|r)=\frac{1}{N}(r-\underline{c})-\int_{\underline{c}}^{r}(1-F(x))^{N-1}F(x)dx-\frac{1}{N}\int_{\underline{c}}^{r}(1-F(x))^{N}dx$$

$$=-\int_{\underline{c}}^{r}(1-F(x))^{N-1}F(x)dx+[-\frac{1}{N}(r-x)(1-F(x))^{N}]_{\underline{c}}^{r}-\frac{1}{N}\int_{\underline{c}}^{r}(1-F(x))^{N}dx$$

$$=-\int_{\underline{c}}^{r}(1-F(x))^{N-1}F(x)dx+\int_{\underline{c}}^{r}(r-x)(1-F(x))^{N-1}f(x)dx$$

$$=[-(r-x)(1-F(x))^{N-1}F(x)]_{\underline{c}}^{r}$$

$$-\int_{\underline{c}}^{r}(1-F(x))^{N-1}F(x)dx+\int_{\underline{c}}^{r}(r-x)(1-F(x))^{N-1}f(x)dx$$

$$=\int_{\underline{c}}^{r}(r-x)(N-1)(1-F(x))^{N-2}F(x)f(x)dx.$$

The last equality uses (14).

B Proof of Proposition 5.3

If for some V_v we obtain $U_v^1(c|r) < 0$ for all $c \in V_v$, it is also true that $U_v^1(l|r) < 0$. In order to prove no credibility, we focus on the lowest cost and prove $U_v^1(l|r) > 0$ for any V_v and r. By (10), the competitive payoff for the vetoer at the lowest cost is given by

$$\pi_v^{1*}(l|r) = (1 - F_r(c_r(\underline{p})))(\underline{p} - c_v(\underline{p})) = \frac{(1 + h - 2l)^2}{4(1 + h - l)}.$$

The supposition of Proposition (5.2), the possible minimum reserve price is $\underline{r} = (1 + h)/2$.

By the above reasoning, substituting $\underline{r}$ into (8) yields the payoff difference:

$$U_v^1(l|\underline{r}) = \pi^m(l|\underline{r}) - \pi_v^{1*}(l|\underline{r})$$
$$= \frac{h^4 - (2 + l)h^3 + 27lh^2 + (-48l^2 + 33l + 10)h + 24l^3 - 48l^2 + 5l + 7}{48(1 + h - l)}.$$

Note that the denominator in the payoff difference is always positive for feasible valuables. Let $\zeta(h; l)$ be the numerator and $\eta(l)$ be $\lim_{h \to l} \zeta(h; l) = l^3 - 15l^2 + 15l + 7$. We will prove that $\zeta(h; l)$ is increasing in h for $l < h \leq 1$ and it is positive. Since $\eta(0) = 7$, $\eta(1) = 8$, and $\eta'(l) = 3l^2 - 30l + 15$, we easily calculate that η attains the local maximum $8(10\sqrt{5} - 21) \simeq 10.89$ at $5 - 2\sqrt{5} \simeq 0.53$. We obtain $\lim_{h \to l} \zeta(h; l) > 0$ for any l.

The derivatives of $\zeta(h; l)$ are

$$\frac{d\zeta}{dh}(h; l) = 4h^3 - 3(2 + l)h^2 + 54lh - 48l^2 + 33l + 10,$$
$$\frac{d^2\zeta}{dh^2}(h; l) = 12h^2 - 6(2 + l)h + 54l.$$

We easily see that $\lim_{h \to l} \frac{d\zeta}{dh}(h; l) = l^3 + 33l + 10 > 0$. We denote the discriminant of the quadratic equation $\frac{d^2\zeta}{dh^2}(h; l) = 0$ by $d(l) = l^2 - 68l + 4$. Let l^* be the smaller solution to $d(l) = 0$. By simple calculation we obtain $l^* = 34 - 24\sqrt{2} \simeq 0.059$. First, we easily see that when the lower bound is large enough for $l > l^*$ in the case of $d(l) < 0$, $\frac{d^2\zeta}{dh^2}(h; l)$ is positive for all h.

Second, when l is lower than l^*, the first order derivative $\frac{d\zeta}{dh}(h; l)$ is decreasing in h in the interval between $(2 + l - \sqrt{d(l)})/4$ and $h^*(l) :=$

$(2 + l + \sqrt{d(l)})/4$. Therefore, in this case the first order derivative attains a local minimum at $h^*(l)$. Substituting $h^*(l)$ in it yields:

$$\frac{d\zeta}{dh}(h^*(l); l) = \frac{1}{8}(-l^3 - 282l^2 + 468l + 72 - d(l)^{\frac{3}{2}}).$$

We easily see that $\frac{d\zeta}{dh}(h^*(0); 0) = 8 > 0$. Since $d(l)$ is decreasing in $l \leq l^*$, we obtain that $-d(l)^{\frac{3}{2}}$ is increasing in the region. Let $\theta(l)$ be $-l^3 - 282l^2 + 468l$. Each solution of the equation $\theta'(l) = -3l^2 - 564l + 468 = 0$ is $-94 \pm 4\sqrt{562}$. We easily compute that the large one ($\simeq 0.83$) is greater than l^* and the small one is negative. We obtain that $\theta'(l) > 0$ and then $\theta(l) \geq 0$ in the region. Therefore, we conclude $\frac{d\zeta}{dh}(h^*(l); l) > 0$ in the region.

Finally, to bring two cases together we obtain that $\frac{d\zeta}{dh}(h; l) > 0$ for any feasible h. We prove that $\zeta(h; l)$ is positive for any h and l with $0 \leq l < h \leq 1$. There does not exist veto set such that $U_v^1(l|r) < 0$ for any l and r.

C   Proof of Proposition 6.1

Proof: The cut-off level h at which the vetoer is indifferent between the collusion and the veto is determined by the following equation:

$$\pi^m(h|r) = (r - h)(1 - F(h))^{N-1}. \tag{15}$$

We first prove that there exists a cut-off cost h.

By Lemma 3.2, let us define

$$g(c) := \pi^m(r|r) + \int_c^r (1 - F(x))^{N-1} dx - (r - c)(1 - F(c))^{N-1}.$$

Then,

$$g'(c) = -(1 - F(c))^{N-1} + (r - c)(N - 1)(1 - F(c))^{N-2} f(c) + (1 - F(c))^{N-1}$$
$$= (r - c)(N - 1)(1 - F(c))^{N-2} f(c) > 0 \quad \text{for } c < r.$$

By (4), $g(\underline{c}) = (r - \underline{c} - T(\underline{c})) - (r - \underline{c}) = -T(\underline{c}) < 0$. Furthermore, $g(r) = \pi^m(r|r) > 0$. Hence, there exists h such that $g(h) = 0$ for $h < r$.

Let the vetoer v's payoff after a veto be π_v^{2*}. We will show that $\pi_v^{2*}(c) \leq \pi^m(c)$ for any c. As in (2), the equilibrium strategy gives

$$
\pi_v^{2*}(c) = \begin{cases}
0 & \text{if } c > r, \\
(r-c)(1-F(h))^{N-1} & \text{if } r \geq c > h, \\
(r-c)(1-F(h))^{N-1} & \\
\quad + \int_c^h (x-c)(N-1)(1-F(x))^{N-2}f(x)dx & \text{if } h \geq c.
\end{cases}
$$

We denote $\pi^m(c) - \pi_v^{2*}(c)$ by $U_v^2(c)$. We obtain that by (4) and (5),

$$
U_v^2(c) = \begin{cases}
\pi^m(r|r) & \text{if } c > r, \\
\pi^m(r|r) + \int_c^r (1-F(x))^{N-1}dx - (r-c)(1-F(h))^{N-1} & \text{if } r \geq c > h, \\
\pi^m(r|r) + \int_c^r (1-F(x))^{N-1}dx - (r-c)(1-F(h))^{N-1} & \\
\quad - \int_c^h (x-c)(N-1)(1-F(x))^{N-1}f(x)dx & \text{if } h \geq c.
\end{cases}
$$

First, we consider the case for $r \geq c > h$:

$$
U_v^2(c) = \pi^m(r|r) + \int_c^r (1-F(x))^{N-1}dx - (r-c)(1-F(h))^{N-1}.
$$

Then,

$$
U_v^{2'}(c) = \begin{cases}
-(1-F(c))^{N-1} + (1-F(h))^{N-1} > 0 & \text{if } c > h \\
0 & \text{if } c = h.
\end{cases}
$$

Since $U_v^2(h) = 0$ by (15), we conclude that $U_v^2(c) \geq 0$ for $r \geq c \geq h$.

Second, we consider the case for $c < h$,

$$
U_v'(c) = -(1-F(c))^{N-1} + (1-F(h))^{N-1} + \int_c^h (N-1)(1-F(x))^{N-2}f(x)dx
$$

$$
= -(1-F(c))^{N-1} + (1-F(h))^{N-1} - [(1-F(c))^{N-1}]_c^h = 0.
$$

Since $U_v^2(h) = 0$, we see that $U_v^2(c) = 0$ for $c \leq h$. The difference of payoffs corresponds to the bold line in Figure 2. Even if the vetoer makes the ratifiers stay out, he only gets his payoff equal to the bid-rigging payoff in this case. Since the vetoer does not strictly gain by vetoing in any case, the candidate veto set is not credible. Since the equilibrium p^* is a weakly

dominant strategy, there is no credible belief and no strategy pertaining to it.

References

Bajari, P. and L. Ye (2003), "Deciding Between Competition and Collusion," *Review of Economics and Statistics*, Vol. 85, No. 4, pp. 971–989.

Bulow, J., J. Roberts (1989), "The Simple Economics of Optimal Auctions," *Journal of Political Economy*, Vol. 97, No. 5, pp. 1060–1090.

Caillaud, B. J. M. and P. Jehiel (1998), "Collusion in Auctions with Externalities," *RAND Journal of Economics*, Vol. 29, No. 4, pp. 680–702.

Cramton, P. and T. R. Palfrey (1990), "Cartel Enforcement with Uncertainty about Costs," *International Economic Review*, Vol. 31, No. 1, pp. 17–47.

Cramton, P. and T. R. Palfrey (1995), "Ratifiable Mechanisms: Learning from Disagreement," *Games and Economic Behavior*, Vol. 10, No. 2, pp. 255–283.

Graham, D. A. and R. C. Marshall (1987), "Collusive Behavior at Single-Object Second-Price and English Auctions," *Journal of Political Economy*, Vol. 95, No. 6, pp. 1217–1239.

Griesmer, J. H., R. E. Levitan, and M. Shubik (1967), "Toward a Study of Bidding Processes Part IV: Games with Unknown Costs," *Naval Research Logistics Quarterly*, Vol. 14, pp. 415–33.

Guesnerie, R. and J. J. Laffont (1984), "A Complete Solution to a Class of Principal-Agent Probems with an Application to the Control of a

Self-Managed Firm," *Journal of Public Economics*, Vol. 25, No. 3, pp. 329–369.

Hinloopen, J. and A. R. Soetevent (2005), "An Experimental Investigation of the Effects of Leniency Programs for Antitrust Enforcement," mimeo.

Holt, C. A., (1980), "Competitive Bidding for Contracts under Alternative Auction Procedures," *Journal of Political Economy*, Vol. 88, No. 3, pp. 433–45.

Kaplan, T.R. and S. Zamir (2007), "Asymmetric First-Price Auctions with Uniform Distributions: Analytic Solutions to the General Case," mimeo.

Lebrun, B. (1999), "First Price Auctions in the Asymmetric N Bidder Case," *International Economic Review*, Vol. 40, No. 1, pp. 125–142.

Levenstein, M. C. and V. Y. Suslow (2006), "What Determines Cartel Success?" *Journal of Economic Literature*, Vol. 44, No. 1, pp. 43–95.

Mailath, G.J. and P. Zemsky (1991), "Collusion in Second Price Auctions with Heterogeneous Bidders," *Games and Economic Behavior*, Vol. 3, No. 4, pp. 467–486.

Maskin, E. and J. Riley (2000a), "Asymmetric Auctions," *Review of Economic Studies*, Vol. 67, No. 3, pp. 413–438.

Maskin, E. and J. Riley (2000b), "Equilibrium in Sealed High Bid Auctions," *Review of Economic Studies*, Vol. 67, No.3, pp. 439–454.

McAfee, R. P. and J. McMillan (1992), "Bidding Rings," *American Economic Review*, Vol. 82, No. 3, pp. 579–599.

Miura, I. (2003), *Economic Analysis of Public Contracts*, Kyushu University

Press (in Japanese).

Myerson, R. B. (1981), "Optimal Auction Design," *Mathematics of Operations Research*, Vol. 6, No. 1, pp. 58–73.

Robinson, M. S. (1985), "Collusion and the Choice of Auction," *RAND Journal of Economics*, Vol. 16, No. 1, pp. 141–145.

Tan, G. and O. Yilankaya (2007), "Ratifiability of Efficient Collusive Mechanisms in Second-Price Auctions with Participation Costs," *Games and Economic Behavior*, Vol. 59, No. 2, pp. 383–396.

Tanno, T. (2008), "Linear Bid in Asymmetric First-Price Auctions," mimeo.