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Cross-Market Platform Competition in Mobile App Economy

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Abstract

The mobile app economy comprises two distinct platform markets through which app developers make revenue: app platform and ad platform markets. App sales are facilitated by app platforms, whereas advertising matching is intermediated by ad platforms. Cross-market platform competition exists, i.e., app and ad platforms compete for developers' revenue sources to earn commissions. In literature and policy debates, however, these platform markets are studied separately. The goal of this study is development of a unified model to examine them jointly. The results provide novel implications for competition policy, which could not be reached without consideration for cross-market platform competition.

JEL Classifications: L1, L4

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1 Introduction

In the mobile application (app) economy, app developers have two revenue channels: app sales and in-app advertising. Developers distribute apps to users through *app platforms* such as Apple App Store and Google Play Store. They can also use *ad platforms* (also called ad networks and ad exchanges), including Google AdMob and others, to match advertisers who place ads on their apps. App developers determine the channel through which to make revenue. Some apps earn from pay-per-download fees (e.g., GoodNote), in-app purchases (e.g., Fortnite), and subscription fees (e.g., Spotify). App platforms collect commissions from those sales. In contrast, some are distributed for free of charge, while generating revenue from in-app advertising (e.g., Facebook and Twitter apps). A fraction of advertising revenue is deprived by ad platforms. One can argue that app and ad platforms compete for developers' revenue sources. Surprisingly, however, these app platform and ad platform markets are discussed separately in the relevant literature and in policy debates (e.g., Australian Competition & Consumer Commission, 2021; Autoriteit Consument & Markt, 2019; Competition & Markets Authority, 2020; Digital Markets Act, 2020). This study was conducted for development of a unified model to examine both platform markets jointly.

1.1 Industry background and motivations

Nowadays, almost everyone has a smartphone or tablet device that can be customized by downloading apps. In fact, the mobile app market has been growing rapidly. Consumer spending on mobile apps in 2020 amounted to USD 143 billion, up 20% from the previous year (App Annie, 2021).

In parallel with the explosive growth of the mobile app market, some observers have warned of the dominant market power of app platforms (Cabral et al., 2021). Consumers are locked into an app store once they buy an Android or Apple device. Therefore, Google and Apple can act as monopolistic gatekeepers to app developers for access to the user base of their operating systems. The gatekeeper position enables them to charge a monopolistic 30% commission on app sales.

In recent years, app platforms have been facing growing pressure to reduce their commission rates. Most notably, in August 2020, Epic Games, an influential game developer, offered their users its own payment method to bypass the Apple and Google payment systems. As a result, Epic was excluded from App Store. Immediately thereafter, Epic filed an antitrust lawsuit against Apple (The Economist, 2020). In response, on January 1, 2021, Apple reduced its commission rate from 30% to 15% for app developers whose annual sales are less than USD 1 million. Google followed the rival by reducing the commission to 15% for the first USD 1 million of revenue developers earn each year.

Many app developers evade payment of the high commission by shifting their revenue channel from app sales to in-app advertising. In fact, as of July 2021, 93.4% of iOS apps and 96.9% of Android apps were delivered for free of charge (Statista Inc., 2021a). Instead, they earn from in-app advertising, which creates a huge market for mobile advertising. Mobile advertising spending worldwide in 2020 amounted to USD 223 billion. It is expected to surpass USD 339 billion by 2023 (Statista Inc., 2021b), which is much greater than the market size of paid apps.

In-app advertising relies on a complex chain of intermediaries, at each stage of which Google holds a near-monopoly position (Competition & Markets Authority, 2020).[1] Since Apple shut down iAd in 2016, Google has been the dominant player in the mobile advertising market, which runs real-time auctions for advertising matching.[2] Competition & Markets Authority (2020) estimated that at least 35% of the value of advertising is captured, on average, by ad platforms.[3] Antitrust concerns are raised not only about Google's dominant position, but also about the lack of market transparency (Jeon, 2021). Also, Cabral et al. (2021) claim that the online advertising market is very opaque. Not all chains of payments can be followed. Some money is "lost" in calculation. Therefore, in most cases, app developers do not know who paid how much for their ad space.

[1]In the U.S., the legal battle against Google's monopolization of online display advertising began in December 2020.

[2]The dominant power enables Google to use data collected from past bids in its digital advertising exchange to favor its own ad-buying system over that of third-party competitors, as reported in the business media (The Wall Street Journal, 2021).

[3]Similarly, Australian Competition & Consumer Commission (2021) indicated that "on average, fees for ad tech services directly involved in the trading and serving of ad impressions made up 28% of advertiser expenditure on display advertising impressions in Australia in 2019."

When taken together, it is noteworthy that distinct platforms collectively constitute a mobile ecosystem. The co-existence of different platform markets is a unique feature of the mobile app economy, which differs greatly from the standard-setting examined in the literature on two-sided markets (Rochet and Tirole, 2003, 2006). Moreover, cross-market platform competition does occur: app and ad platforms compete for app developers' revenue sources. To elucidate the complex landscape of the mobile ecosystem and take the debate one step further, it is vitally necessary to consider both platform markets together.

1.2 Contributions

Toward that end, a benchmark model with an app platform and an ad platform is examined, in which the app platform intermediates between users and developers of apps, whereas the ad platform intermediates between ad-funded apps and advertisers. They charge an ad valorem commission for their intermediation services. A lower ad commission (respectively, app commission) encourages app developers to make revenue from in-app advertising (resp., app sales) through the ad platform (resp. app platform).

After observing both app and ad commissions, each app developer decides the app price and the amount of ads to be displayed. App developers are heterogeneous both in terms of the benefit they create to advertisers and for the nuisance cost they impose upon users. Developers of apps that generate a higher advertising benefit compared to the nuisance cost are more likely to adopt an ad-funded model (e.g., Twitter), whereas others will deploy a pay-per-download model (e.g., GoodNotes).

The equilibrium allocation between paid apps and ad-funded apps is characterized and compared to the socially optimal one that maximizes social welfare. Results show that, for welfare maximization, app and ad commissions should be set at the same level. However, in equilibrium, the resulting app commission might be lower or higher than the ad commission, depending on the circumstances. Analyses further demonstrate that, as apps are designed increasingly to create an advertiser benefit greater than the nuisance cost they impose upon users, the app platform is increasingly likely to charge a commission that is lower than the ad commission set by the ad platform. This is true because, if not doing so, then the app platform fails to attract a sufficient number of app developers. The lower app commission results in an oversupply of paid apps in terms of social welfare.

This misallocation (i.e., oversupply of paid apps) might be amplified further by the recent movement of putting pressure on app platforms to reduce their commissions. Actually, Apple and Google responded to this social pressure by reducing their commission rate from 30% to 15%, whereas at least 35% of the value of in-app advertising is captured by ad platforms (Competition & Markets Authority, 2020). This trend might lead some developers to shift their revenue channel from in-app advertising to greater emphasis on pay-per-download fees, in-app purchases, and subscription fees, implying that users will pay more for mobile apps (Sokol and Zhu, 2021).

Additionally, a simple sufficient condition for which paid apps are oversupplied is derived: If the ratio of paid apps is less than 50%, then they are oversupplied in terms of social welfare. One can infer that the current situation seems to satisfy this condition because over 90% of apps are free of charge. It is also worth emphasizing that this sufficient condition depends only on the ratio of paid apps, not on the respective values of app and ad commissions. Therefore, even if the advertising market becomes more opaque so that no one can estimate ad commissions in the future, the condition is expected to work well as long as we can observe the ratio of paid apps.

Next, the benchmark model is extended to incorporate competition among ad platforms. Currently, although Google has a near-monopoly position in the advertising market, several competitors exist (e.g., InMobi and Index Exchange). Moreover, until 2016, Apple used to operate its ad platform division: iAd. Accordingly, two specific situations are examined for the ownership structure of competing ad platforms. The first is that competing ad platforms are independent of the app platform. This situation corresponds to the current iOS app economy, in which ad platforms are independent of Apple controlling the monopolistic app store for iOS users. The second situation is that one competing ad platform is operated jointly by the app platform. This situation reflects the current Android OS app economy, where Google runs both the Play Store and AdMob, and the past iOS app economy, in which Apple had operated iAd in addition to its App Store.

The extended analyses demonstrate that, irrespective of the ownership structure, ad platform competition engenders a lower ad commission, which also compels the monopoly app platform to reduce its app commission because of cross-market platform competition. The former reduction is greater than the latter one. Consequently, an undersupply of paid apps can occur.

In the case of cross-market platform integration, fierce ad platform competition not only engenders a low ad commission; it also forces the integrated platform to charge a low app commission. Therefore, results show that the integrated platform can benefit from a shutdown of its ad platform division that is involved in fierce competition. In fact, doing so reduces the intensity of ad platform competition and therefore enables it to charge a higher app commission eventually. This finding might explain why Apple terminated its iAd service, which had faced fierce direct competition with Google AdMob.

The model developed for this study is useful to elucidate the *past* and *present* of the mobile app economy. Moreover, it is expected to facilitate the formulation of better predictions about the *future*, e.g., the consequences of Apple's new iOS policy (AppTrackingTransparency, ATT), as discussed later. Results provide important implications for competition policy that could not be reached if we had not considered cross-market platform competition.

1.3 Related literature

This paper contributes to the burgeoning literature of theoretical studies of two-sided markets (e.g., Rochet and Tirole, 2003, 2006; Armstrong, 2006; Weyl, 2010) in which app and ad platform markets have been studied separately. On the one hand, app platforms are considered to provide a marketplace where buyers and sellers transact directly, resembling Amazon and eBay. Existing studies examine the microfoundations for buyer–seller transactions (e.g., Hagiu, 2009; Karle et al., 2020). Platforms of this type earn commissions from sales at the marketplaces, whereas sellers are allowed to charge prices directly to buyers. This arrangement is designated as an agency model (Edelman and Wright, 2015; Johnson, 2017). The present paper differs from these existing studies in that sellers (i.e., app developers in this paper) are allowed to have another revenue channel, i.e., advertising revenue. The existence of this revenue channel induces some sellers to set prices at zero. Thereby, platforms cannot earn any commission from them, which is in sharp contrast to results obtained from existing studies.

On the other hand, ad platforms (also called ad networks) are examined in recent work reported by D'Annunzio and Russo (2020).[4] In their model, an ad network intermediates between publishers (i.e., app developers in this paper) and advertisers. The publishers are regarded exogenously as fully ad-funded platforms, i.e., they are not allowed to charge prices. At this point, the present paper differs from theirs in that app developers are allowed to charge prices to users, which leads to the necessity for considering the presence of app platforms.

Additionally, one can consider that another type of platform exists in this study. Ad-funded apps are also platforms that enable advertisers to catch the attention of app users. This type of platform has been studied in the literature related to media platforms (e.g., Anderson and Coate, 2005). A feature of this literature is the presence of negative indirect network externalities from advertisers to users. Recent studies allow for multi-homing by users (Ambrus et al., 2016; Athey et al., 2018) and the use of ad blockers (Anderson and Gans, 2011; Despotakis et al., 2021). Nevertheless, neither study allows for the presence of advertising intermediaries (i.e., ad platforms in this paper) that enable ad-sponsored platforms to meet their sponsors.

The remainder of this paper is structured as follows. The benchmark model consisting of an app platform and an ad platform is described in Section 2 and is analyzed in Section 3. In Section 4, ad platform competition is considered with different ownership structures. Section 5 presents conclusions of this study.

[4]Little has been uncovered about details of complex chains of multilayered intermediaries in online advertising markets. One can refer to an excellent survey reported by Choi et al. (2020). More recently, Decarolis and Rovigatti (2021) report their empirical investigation of the effects of intermediary concentration on the allocation of revenues in the chain.

2 Benchmark Model

This section presents a benchmark model with an app platform and an ad platform. The app platform charges an ad valorem commission of r for intermediation of app sales, whereas the ad platform sets an ad valorem commission of τ for intermediation of advertising matching (so-called "ad tech tax").

There are N users, labeled as $i \in \{1, \ldots, N\}$, each of whom is ex ante identical. There also exists a continuum of app developers, labeled as $\rho \in [0, \overline{\rho}]$. We use ρ to denote the type of apps, which follows a distribution function $G(\rho)$ with positive density function $g(\rho)$. App developer ρ decides app price $p_\rho \geq 0$ and ad space $A_\rho \geq 0$. The total mass of apps is assumed to be constant and normalized to one.

What surplus user i obtains from the use of app ρ is given as $\varepsilon_{i\rho} - p_\rho - \delta(\rho)A_\rho$, where $\varepsilon_{i\rho}$ is viewed as a match value expressing the stand-alone benefit of app ρ to user i, and where $\delta(\rho)$ denotes the extent of users' disutility created by a unit of ads displayed in app ρ. We assume that $\delta(\rho)$ is positive[5] and decreasing in $\rho \in [0, \overline{\rho}]$, i.e., $\delta(\rho) > 0$ and $\delta'(\rho) < 0$. The latter means that the app with higher ρ causes smaller advertising disutility to users.

Users draw a value of $\varepsilon_{i\rho}$ from distribution function $F(\varepsilon)$, which is independent and identically distributed across all apps. User i purchases app ρ if and only if $\varepsilon_{i\rho} - p_\rho - \delta(\rho)A_\rho \geq 0$.[6] Consequently, the demand of app ρ is given as $D_\rho(p_\rho, A_\rho) = N\left\{1 - F(p_\rho + \delta(\rho)A_\rho)\right\}$. For the existence and uniqueness of the equilibrium, we impose the Monotone Hazard Rate assumption, i.e., $\frac{f(x)}{1-F(x)}$ is monotone non-decreasing,[7] where $f(\cdot) = F'(\cdot)$. Consumer surplus is computed as follows.

$$CS = N \int_0^{\overline{\rho}} \int_{p_\rho + \delta(\rho)A_\rho}^{\infty} \left\{\varepsilon - p_\rho - \delta(\rho)A_\rho\right\} dF(\varepsilon) dG(\rho) \tag{1}$$

[5]Ghose and Han (2014) empirically demonstrate that the presence of in-app ads reduces the demand for mobile apps.

[6]That is, all apps are independent. No competition exists between them. A similar approach is adopted by Etro (2021).

[7]The assumption is satisfied for the Uniform, Normal, Pareto, Logistic, Exponential, and any distribution with non-decreasing density.

Next, the profit of app ρ is expressed as

$$\pi_\rho = \underbrace{(1-r)p_\rho D_\rho(p_\rho, A_\rho)}_{\text{from app sales}} + \underbrace{(1-\tau)\beta(\rho)A_\rho D_\rho(p_\rho, A_\rho)}_{\text{from in-app advertising}} \tag{2}$$

where $\beta(\rho) > 0$ denotes the per-user advertising revenue generated from a unit of ads displayed in app ρ (Choi and Jeon, 2020). One can consider that $\beta(\rho)$ is equal to the price paid by a winning bidder of the advertising auction intermediated by the ad platform.[8] We assume that $\beta(\rho)$ is an increasing function in $\rho \in [0, \overline{\rho}]$, i.e., $\beta'(\rho) > 0$. Expressed in words, an app with higher ρ is more valuable for advertisers.

Assumptions of $\beta'(\rho) > 0$ and $\delta'(\rho) < 0$ imply that an app with higher ρ creates greater advertising benefits for advertisers while generating smaller nuisance costs for users. In other words, an app with high ρ is designed to have a strong affinity for advertising. One can infer that, for example, Twitter has a high ρ value because the ads displayed on Twitter are not too annoying, while providing useful information to users sometimes. In contrast, GoodNotes might have a low value of ρ. If an ad showed up on your display while you were reading a paper for refereeing, then it would be very annoying and you would not think well of the product being advertised.

The app developer profit, as presented in Equation (2), derives from both app sales and in-app advertising. A share r of sales revenue is taken by the app platform, whereas a share τ of advertising revenue is deprived by the ad platform. Given r and τ, app developer ρ chooses a pair of (p_ρ, A_ρ) to maximize its profit. The combination of p_ρ and A_ρ represents their business model. When $p_\rho > 0$ and $A_\rho = 0$, app ρ is said to adopt a pay-per-download model. In contrast, when $p_\rho = 0$ and $A_\rho > 0$, app ρ is said to employ an ad-funded model. App developer surplus can be computed as $ADS = \int_0^{\overline{\rho}} \pi_\rho dG(\rho)$.

The app platform's and ad platform's profits are calculated, respectively, as

[8] Google switched from second-price to first-price auctions in 2019 (Gordon et al., 2021). According to AdExchanger (2019), Google was the latest switcher among major ad exchanges. Other ad exchanges have adopted first-price auctions since 2017.

follows.

$$\Pi_{App} = \int_0^{\overline{\rho}} r p_\rho D_\rho dG(\rho) \tag{3}$$

$$\Pi_{Ad} = \int_0^{\overline{\rho}} \tau\beta(\rho) A_\rho D_\rho dG(\rho) \tag{4}$$

Social welfare is defined as the sum of consumer surplus and profits of all firms as $W = CS + ADS + \Pi_{App} + \Pi_{Ad}$.

The timing of the game is the following. In Stage 1, the app and ad platforms respectively charge commission rates of r and τ. In Stage 2, app developers choose a combination of (p_ρ, A_ρ). The equilibrium concept is the subgame perfect equilibrium. We solve the game using backward induction.

2.1 Discussion of modeling assumptions

In the benchmark model, the two distinct platforms are considered, each of which has no direct competitor *within* the market while they compete *across* the markets. This setting is apparently relevant in the current configuration of the mobile app economy.

First, the model includes the assumption of a monopoly app platform. This assumption stems from the fact that users visit a given app store according to the operating system of their devices, which implies that no direct competition exists between Apple App Store and Google Play Store. In other words, each app platform faces a fixed market size of users (i.e., N) and acts as a monopolistic gatekeeper for access to them.

One might wonder that Apple and Google compete in selling smartphone devices. However, this competition is considered to occur far away from the app platform market considered in the present model. When choosing a smartphone to buy, people consider various factors (e.g., brand reputation, battery capacity, and screen size). The variety of available apps is of course important, but it is not a crucially important factor (Geradin and Katsifis, 2020). In fact, most app developers work on multiple operating systems (i.e., multi-homing), implying that the difference in available app variety between app platforms is not great (Autoriteit Consument & Markt, 2019).

Therefore, we assume a monopoly app platform for this study.[9]

Next, the benchmark model also assumes a monopoly ad platform because Competition & Markets Authority (2020) reports that Google holds a strong, near-monopoly position at each stage of the complex chain of advertising intermediaries. That said, indeed, competitors exist in the market. Therefore, in Section 4, we extend the model in a way that includes competition between ad platforms.

Additionally, in the model, each app developer is presumed to have an exogenously fixed type ρ. One might consider that changes in app and ad commissions would make app developers adjust their type, for example, through changes in the app design. Such long-term effects of cross-market platform competition are not addressed in this study.

3 Analysis

This section presents derivation of the equilibrium of the benchmark model. The model is solved backwardly.

In Stage 2, given r and τ, app developer ρ chooses p_ρ and A_ρ to maximize the following profit.

$$\pi_\rho(p_\rho, A_\rho) = (1-r)\left(p_\rho + \underbrace{\frac{(1-\tau)\beta(\rho)}{1-r}}_{\text{effective marginal advertising revenue per user}} A_\rho\right)D_\rho \tag{5}$$

The "effective marginal advertising revenue per user" can be regarded as a negative marginal cost for the app developer (Choi and Jeon, 2020), which plays an important role in determining the developer's business model, as presented later. The derivatives of π_ρ with respect to p_ρ and A_ρ are given as presented below.

$$\frac{\partial \pi_\rho}{\partial p_\rho} = (1-r)\left\{D_\rho - \left(p_\rho + \frac{(1-\tau)\beta(\rho)}{1-r}A_\rho\right) \cdot N \cdot f\left(p_\rho + \delta(\rho)A_\rho\right)\right\} \tag{6}$$

[9]Moreover, in practice, Google adopts an open-platform strategy, unlike Apple. Although this issue is far beyond the scope of this study, one can refer to recent relevant work by Etro (2021). He models competition between a device-funded platform (e.g., Apple) and an ad-funded platform (e.g., Google). However, unlike this paper, the presence of ad platforms is not examined in his model.

$$\frac{\partial \pi_\rho}{\partial A_\rho} = (1-r)\delta(\rho)\left\{\frac{1-\tau}{1-r}\frac{\beta(\rho)}{\delta(\rho)}D_\rho - \left(p_\rho + \frac{(1-\tau)\beta(\rho)}{1-r}A_\rho\right)\cdot N \cdot f\left(p_\rho + \delta(\rho)A_\rho\right)\right\} \tag{7}$$

One can note that no pair of (p_ρ, A_ρ) exists which equalizes Equations (6) and (7) to be zero simultaneously, unless $\frac{1-\tau}{1-r}\frac{\beta(\rho)}{\delta(\rho)} = 1$.

The optimal strategy varies depending on whether $\frac{1-\tau}{1-r}\frac{\beta(\rho)}{\delta(\rho)}$ is less than or greater than 1, which condition depends on whether the effective marginal advertising revenue per user $\frac{(1-\tau)\beta(\rho)}{1-r}$ is less than or greater than the marginal advertising disutility $\delta(\rho)$. We let $\hat{\rho}(r,\tau)$ solve $\frac{(1-\tau)\beta(\hat{\rho})}{1-r} = \delta(\hat{\rho})$. One can ensure that $\hat{\rho}$ is unique because of assumptions of $\beta'(\rho) > 0$ and $\delta'(\rho) < 0$. Consequently, for $\rho < \hat{\rho}$, it holds that $\frac{(1-\tau)\beta(\rho)}{1-r} < \delta(\rho)$.

The following proposition describes the optimal strategy of app developers in Stage 2. Detailed proofs are given in the Appendix.

Proposition 1. *For app developers with $\rho < \hat{\rho}(r,\tau)$, the optimal strategy is characterized as $(p_\rho, A_\rho) = (p^+, 0)$, where p^+ solves $1 - F(p^+) = p^+ f(p^+)$. For the remaining developers, the optimal strategy is $(p_\rho, A_\rho) = (0, A^+(\rho))$, where $A^+(\rho) = p^+/\delta(\rho)$. The resulting demand is the same across all apps, i.e., $D_\rho = N\{1 - F(p^+)\} \equiv D^+$ for all $\rho \in [0, \overline{\rho}]$.*

This proposition shows that a threshold value $\hat{\rho}(r,\tau)$ exists such that app developers with $\rho < \hat{\rho}$ adopt the pay-per-download model.[10] That is, they place no ad space and earn revenues completely from fees paid by users because their apps are less compatible with advertising. We designate them as paid apps. In contrast, the others with $\rho > \hat{\rho}$ deploy the ad-funded model. Their apps are distributed free of charge. The profits derive solely from advertising revenues. We designate them as ad-funded apps.

Moreover, the following corollary is obtained.

Corollary 1. *The threshold value $\hat{\rho}(r,\tau)$ decreases with r and increases with τ. Formally, $\frac{\partial \hat{\rho}(r,\tau)}{\partial r} < 0$ and $\frac{\partial \hat{\rho}(r,\tau)}{\partial \tau} > 0$ hold.*

[10] Kawaguchi et al. (2021) estimated that the disutility of watching one unit of advertisement is, on average, JPY 56.4 for games and JPY 14 for other applications. Moreover, the data used in their study exhibit that the ratio of apps showing some advertisements is lower in games than in other apps.

This corollary implies that a reduction in the app commission rate encourages app developers to adopt the pay-per-download model. This implication is consistent and complementary with the finding of Kawaguchi et al. (2021), who built a structural model of the mobile app industry and then estimated it using data from Japan. Their counterfactual analysis demonstrates that an exogenous reduction in app commission increases app download prices and decreases the amount of advertisements, implying that an app commission reduction incentivizes app developers to shift their revenue channel from in-app advertising toward app sales, as shown in Corollary 1.[11]

Corollary 1 also indicates that two platforms of different types compete for app developers' revenue sources. The app platform wants more app developers to adopt the pay-per-download model, whereas the ad platform tempts them to choose the ad-funded model. Given commissions r and τ, app developers decide which revenue source upon which they will rely. It is important to ascertain whether the resulting allocation of apps between the two business models is distorted from the optimal allocation in terms of social welfare. If distorted, which type of app is supplied excessively? What kind of policy intervention is desirable?

To address those questions, we consider a welfare maximization problem before addressing the equilibrium outcome of cross-market platform competition. Specifically, we allow a policymaker to choose both r and τ to maximize social welfare.[12] With the results of Proposition 1, social welfare is computed as

$$W(r,\tau) = N\int_0^{\overline{\rho}}\int_{p^+}^{\infty} \varepsilon \ dF(\varepsilon)dG(\rho) + p^+D^+\int_{\hat{\rho}(r,\tau)}^{\overline{\rho}} \frac{\beta(\rho)-\delta(\rho)}{\delta(\rho)}dG(\rho), \quad (8)$$

where the first term is independent of r and τ. Consequently, the welfare maximization problem is equivalent to solving the maximization problem for the second term of Equation (8). The second term is maximized when $\hat{\rho}(r,\tau)$ is set so that $\beta(\hat{\rho}) = \delta(\hat{\rho})$ holds, or equivalently, $r = \tau$ (see Appendix for details). The following proposition presents the welfare maximization result.

[11] It is noteworthy that, in their structural model, ad platforms do not appear as strategic players that determine ad commissions endogenously, unlike the present paper.

[12] Maximization of consumer surplus is also a notable phenomenon. With the results of Proposition 1, consumer surplus is computed as $CS = N\int_0^{\overline{\rho}}\int_{p^+}^{\infty}(\varepsilon - p^+)\, dF(\varepsilon)dG(\rho)$, which is independent of r and τ.

Proposition 2. *If a policymaker chooses r and τ to maximize social welfare, then these commission rates are set to be the same, i.e., $r = \tau$.*

This proposition shows that the policymaker favors the same commission rate to be set by the two distinct platforms. In other words, if either is lower than the other, a misallocation occurs. For example, if the app platform charges a commission rate lower than that of the ad platform (i.e., $r < \tau$), then threshold $\hat{\rho}$ becomes greater than the socially optimal one, leading to an oversupply of paid apps. That outcome implies that a socially excessive number of app developers choose the pay-per-download model, although some of them might generate a positive net surplus of advertising (i.e., $\beta(\rho) > \delta(\rho)$ for some ρ). The reverse occurs when $r > \tau$.

Next, we examine the equilibrium of the game where two distinct platforms compete. Given the results of Proposition 1, the profit functions of the app platform and ad platform are rewritten as shown below.

$$\Pi_{App}(r,\tau) = \int_0^{\hat{\rho}(r,\tau)} r p^+ D^+ dG(\rho) \tag{9}$$

$$\Pi_{Ad}(r,\tau) = \int_{\hat{\rho}(r,\tau)}^{\overline{\rho}} \tau \beta(\rho) \frac{p^+}{\delta(\rho)} D^+ dG(\rho) \tag{10}$$

The first-order condition for profit maximization of the app platform is

$$\frac{\partial \Pi_{App}}{\partial r} = p^+ D^+ \left(G(\hat{\rho}(r,\tau)) + r g(\hat{\rho}(r,\tau)) \frac{\partial \hat{\rho}(r,\tau)}{\partial r} \right) = 0. \tag{11}$$

We use $r = r(\tau)$ to denote the best response strategy which solves Equation (11).

Similarly, the first-order condition for profit maximization of the ad platform is given as follows.

$$\frac{\partial \Pi_{Ad}}{\partial \tau} = p^+ D^+ \left(\int_{\hat{\rho}(r,\tau)}^{\overline{\rho}} \frac{\beta(\rho)}{\delta(\rho)} dG(\rho) - \tau \frac{\beta(\hat{\rho}(r,\tau))}{\delta(\hat{\rho}(r,\tau))} g(\hat{\rho}(r,\tau)) \frac{\partial \hat{\rho}(r,\tau)}{\partial \tau} \right) = 0 \tag{12}$$

By $\tau = \tau(r)$, we denote the best response strategy which solves Equation (12).

To ensure the existence and uniqueness of the equilibrium, we impose the following assumption:

$$\frac{g'(\hat{\rho})}{g(\hat{\rho})} + \frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} < \frac{(1-\tau)\beta''(\hat{\rho}) - (1-r)\delta''(\hat{\rho})}{(1-\tau)\beta'(\hat{\rho}) - (1-r)\delta'(\hat{\rho})} < \frac{g'(\hat{\rho})}{g(\hat{\rho})} + 2\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})}, \quad (13)$$

where $\hat{\rho}$ is evaluated at $(r, \tau) \in [0, 1]^2$. This condition requires that functions $\beta(\cdot)$ and $\delta(\cdot)$ not be too convex and not be too concave. Moreover, it is worth noting that Condition (13) guarantees that the second-order conditions for maximization are satisfied and that r and τ are strategic complements. More details are provided in the proof of Proposition 3 in the Appendix.

Proposition 3. *Under Condition* (13)*, there exists a unique equilibrium* (r^*, τ^*) *that satisfies Equations* (11) *and* (12)*. In equilibrium, paid apps are oversupplied in terms of social welfare if and only if* $r^* < \tau^*$*. Letting* $\rho^* = \hat{\rho}(r^*, \tau^*)$*, the necessary and sufficient condition for* $r^* < \tau^*$ *is written as* $\beta(\rho^*) > \delta(\rho^*)$*, or equivalently*

$$\frac{\delta(\rho^*)}{\beta(\rho^*)} \int_{\rho^*}^{\overline{\rho}} \frac{\beta(\rho)}{\delta(\rho)} dG(\rho) > G(\rho^*). \quad (14)$$

We use '$*$' to denote the equilibrium outcome of the benchmark model.

Proposition 3 presents the necessary and sufficient condition for which the number of paid apps is greater than the socially optimal level. If the actual commission rates are publicly observable, then one can use the simplest condition $r^* < \tau^*$. In 2021, Apple and Google reduced their commission from 30% to 15% partially. At the time, Competition & Markets Authority (2020) estimated that ad networks capture at least 35% of the value of advertising (i.e., $r = 0.15 < 0.35 = \tau$). This evidence suggests that paid apps are being oversupplied in the current mobile app economy. That is, the reduction in app commission might worsen social welfare by exacerbating the oversupply of paid apps. Kawaguchi et al. (2021) report a similar conclusion by showing that a reduction in app commission from 30% can be detrimental to social welfare.[13]

Moreover, even if one cannot observe or estimate the actual values of commission rates, Condition (14) enables identification of whether paid apps are oversupplied or

[13]They also argue that, if applied exclusively on game apps, then the optimal commission rate is approximately 12–15%.

undersupplied. The right-hand side of the condition represents the ratio of paid apps in equilibrium. The left-hand side measures the ratio of ad-funded apps counted with weighting by the ratio of the advertising benefit to advertising disutility. The inequality is more likely to hold as the equilibrium ratio of paid apps declines.

However, to use Condition (14), one must estimate functions $\beta(\cdot)$ and $\delta(\cdot)$, which is not an easy task. Instead, one can use a simple sufficient condition for Condition (14), as presented below.

Corollary 2. *If the ratio of paid apps is less than half (i.e., $G(\rho^*) < 1/2$), then paid apps are oversupplied in terms of social welfare.*

In fact, as of July 2021, 93.4% of iOS apps and 96.9% of Android apps were made available free of charge (Statista Inc., 2021a), i.e., the ratio of paid apps is clearly less than half. Considered along with the preceding discussion, this evidence can strengthen the claim that paid apps are currently being oversupplied in terms of social welfare. Therefore, it might not be a good movement that app platforms are under pressure for reduction of their commissions.

Based on these results, one would recognize the importance of examining app platform and ad platform markets jointly when considering good policymaking.

4 Extensions: Ad Platform Competition

In earlier sections, following the fact that Google holds a strong, near-monopoly position in the mobile advertising market (Competition & Markets Authority, 2020), the model with a monopoly ad platform is examined. Indeed, some competitors exist, including InMobi, Index Exchange, and others. Especially, before Apple terminated its iAd service in 2016, fierce competition had taken place between Google and Apple. The presence of this strong competitor was one reason why the U.S. Federal Trade Commission (FTC) agreed to allow Google's purchase of AdMob.

Therefore, in this section, we extend the benchmark model in a way that incorporates competition among ad platforms. This extension can support our re-assessment of the consequences of Apple's termination of iAd. Specifically, two cases are investigated. Section 4.1 presents an examination of a model with an app platform and two independent ad platforms. Next, in Section 4.2, we allow for cross-market platform integration, i.e., the app platform integrates a competing ad platform, as Google does and Apple used to do.

4.1 Independent ad platforms

We consider that two homogeneous ad platforms ($j = 1, 2$) are competing in the advertising market. Ad platform j charges a commission rate τ_j. App developers choose an ad platform with a lower commission rate (i.e., perfect competition) if they place some ads.[14] Consequently, the profit of ad platform j is given as shown below.

$$\Pi^j_{Ad} = \begin{cases} \int_0^{\overline{\rho}} \tau_j \beta(\rho) A_\rho D_\rho dG(\rho) & \text{if } \tau_j < \tau_k \\ \frac{1}{2} \cdot \int_0^{\overline{\rho}} \tau_j \beta(\rho) A_\rho D_\rho dG(\rho) & \text{if } \tau_j = \tau_k \\ 0 & \text{if } \tau_j > \tau_k \end{cases} \tag{15}$$

The other aspects of the setting of the game remain unchanged. We derive the equilibrium of the game with ad platform competition. Then we compare it with that of the benchmark model.

Letting $\tau = \min\{\tau_1, \tau_2\}$, given (r, τ), the app developers' decisions made in Stage 2 remain unchanged, i.e., the result of Proposition 1 still holds.

In Stage 1, standard Bertrand competition occurs between ad platforms. That is, competing ad platforms have an incentive to undercut the rival's commission, resulting in $\tau_1^{**} = \tau_2^{**} = 0$ in equilibrium. We use '$**$' to denote the equilibrium outcome of the model with ad platform competition.[15]

Therefore, given $\tau^{**} = 0$, the app platform sets its commission rate at $r^{**} = r(0)$, which is strictly less than r^* because of strategic complementarity (i.e., $r'(\tau) > 0$). That is, it follows that $0 < r^{**} = r(0) < r(\tau^*) = r^*$.

Moreover, in equilibrium, $r^{**} > \tau^{**}$ holds, which in turn implies that paid apps are undersupplied in terms of social welfare, according to Proposition 3.

The following proposition summarizes the preceding analysis.

[14]Currently, it is difficult for advertisers to multi-home to multiple ad intermediaries. For more details, one can refer to a report by Jeon (2021).

[15]The results do not differ even if more than two ad platforms are competing.

Proposition 4. *With perfect competition between ad platforms, they charge* $\tau_1^{**} = \tau_2^{**} = 0$*. The app platform sets its commission rate at* $r^{**} = r(0)$*, which is strictly lower than that of the benchmark model (i.e.,* $r^{**} < r^*$*). Moreover, in equilibrium, paid apps are undersupplied in terms of social welfare.*

Perfect competition in the ad platform market results in the lowest ad commission, which induces the app platform to respond by decreasing its app commission because of cross-market platform competition. However, the former reduction is greater than the latter one. Therefore, some app developers who are close to the threshold type decide to change their business model towards the ad-funded model from the pay-per-download model. As a result, an undersupply of paid apps can occur.

More generally, one can imagine a comprehensive situation using a conduct parameter that represents the degree of competition in the ad platform market, in which the benchmark and current models are included as two extreme cases. It is implied that, as the conduct parameter leans toward intensifying the ad platform competition, an undersupply of paid apps is more likely to happen.

4.2 Cross-market platform integration

Inspired by the fact that Google operates both an app platform (Play Store) and an ad platform (AdMob) and that Apple also used to operate an ad platform (iAd) in addition to its app platform (App Store), here we examine the situation in which one of competing ad platforms (say, ad platform 1 with no loss of generality) is jointly operated by the app platform. Consequently, the integrated platform chooses r and τ_1 to maximize its joint profit $\Pi_{App} + \Pi_{Ad}^1$.

As above, ad platform 2 has an incentive to undercut the rival's commission. Moreover, even with integration, the integrated platform remains to have the undercutting incentive. Consequently, as a result of the standard Bertrand competition, the equilibrium ad commission is equal to zero (i.e., $\tau_1^{**} = \tau_2^{**} = 0$), as in the model with independent ad platforms. Accordingly, the app commission also remains unchanged (i.e., $r = r^{**}$).

Proposition 5. *Even if the app platform integrates either one of the two ad platforms, the equilibrium outcome remains the same as that of Proposition 4.*

This proposition implies that the integrated platform must lower its app commission because it has an ad platform division, although it faces no direct competitors in the market of app intermediation.

One might infer that the integrated platform can benefit from shutting down its ad platform division, as Apple did, because doing so enables it to raise its app commission. To confirm this point, we compare the joint profit of the integrated platform $\Pi_{App}(r^{**}, \tau^{**}) + \Pi^1_{Ad}(r^{**}, \tau^{**})$ with the profit when shutting down the ad platform division $\Pi_{App}(r^*, \tau^*)$.

Proposition 6. *The integrated platform actually benefits from the shutdown of its ad platform division. Formally, it follows that* $\Pi_{App}(r^*, \tau^*) > \Pi_{App}(r^{**}, \tau^{**}) + \Pi^1_{Ad}(r^{**}, \tau^{**})$.

This proposition shows that the integrated platform has an incentive to relinquish its ad platform division when it is involved in fierce competition. In doing so, competition in the ad platform market weakens, which engenders a higher ad commission. An increase in the ad commission subsequently enables the remaining app platform division to charge a higher app commission and to earn greater profits. This result might explain why Apple pulled its ad platform division out of the highly competitive market.

It is noteworthy that, although Proposition 6 shows that the shutdown is always beneficial, this strong result partially relies on the assumption of perfect competition in the ad platform market. That is, no direct losses occur from the shutdown because the ad platform division generates no profits under perfect competition. The shutdown yields some direct losses if the ad platform market is imperfectly competitive. However, the result of Proposition 6 would still hold, unless the direct losses exceed the positive gains associated with reduction in cross-market platform competition.

5 Concluding Remark

The model of cross-market platform competition presented herein provides a unified framework to view the complex landscape of the mobile app economy. This study offers new insights into competition policy, providing results that could not be obtained without consideration of a model with cross-market platform competition

A simple and direct message is that the allocation between paid apps and ad-funded apps is optimized in terms of social welfare when the same commission rate is set across app platform and ad platform markets. If the app commission is lower (resp. higher) than the ad commission, then paid apps are likely to be oversupplied (resp. undersupplied) compared to the socially optimal level. These results and the related discussions indicate that, at *present*, the market might be experiencing an oversupply of paid apps.

Moreover, the model enables re-assessment of *past* events such as Apple's termination of its iAd service in 2016. Results provide a reasonable explanation for why Apple did so. By shutting down its ad platform division that had been involved in fierce competition with Google, Apple has succeeded in keeping a high commission in App Store (i.e., so-called Apple Tax of 30%).

One can also use the model and results to make predictions for the *future* of changeable market environments. For example, in 2021, Apple started its new policy for consumer privacy, namely "AppTrackingTransparency" (ATT), which requires app developers to receive the user's permission when tracking information that is necessary for providing personalized advertising.[16] This policy change is expected to diminish the value of mobile advertising (Sokol and Zhu, 2021), which signals a downward shift of $\beta(\cdot)$ in the model of this paper. If function $\beta(\rho)$ shifts downward, then given app and ad commissions being fixed, the threshold value $\hat{\rho}$ increases. That outcome implies that some developers change their business model from ad-funded to pay-per-download models. This migration of developers' revenue sources is beneficial to Apple, although it would exacerbate the oversupply of paid apps and thereby increase consumer spending on mobile apps.

It is also noteworthy that the result showing that app and ad commissions should be set at the same level for welfare maximization remains unchanged even if functions $\beta(\cdot)$ and $\delta(\cdot)$ change. In other words, cross-market price competition between app and ad platforms can be assessed independently of changes in the policy and design of platforms.

[16]Source: `https://developer.apple.com/app-store/user-privacy-and-data-use/`

The modeling framework developed in this study is expected to be a first step toward elucidating the mobile ecosystem. The framework should be extended in several ways in future studies. Incorporation of oligopolistic competition between app developers would be beneficial. In so doing, considering the presence of some killer apps is also interesting. Moreover, one can address the issue of first-party selling of apps by app platforms themselves (e.g., Apple Music) and evaluate the competitive effects of self-preferencing by app platforms facing cross-market platform competition.

Appendix: Proofs

Proof of Proposition 1 First, we consider the case of $\frac{1-\tau}{1-r}\frac{\beta(\rho)}{\delta(\rho)} < 1$. A pair of (p_ρ, A_ρ) that satisfies $\frac{\partial \pi_\rho}{\partial A_\rho} = 0$ yields $\frac{\partial \pi_\rho}{\partial p_\rho} > 0$. Such a pair can never be optimal because a marginal increase in p_ρ increases the profit. In contrast, a pair of (p_ρ, A_ρ) that satisfies $\frac{\partial \pi_\rho}{\partial p_\rho} = 0$ yields $\frac{\partial \pi_\rho}{\partial A_\rho} < 0$. Such a pair can be optimal if $A_\rho = 0$ because A_ρ is assumed to take a non-negative value. Given $A_\rho = 0$, the optimal choice of p_ρ is to solve $\frac{\partial \pi_\rho(p_\rho, 0)}{\partial p_\rho} = 0$, or equivalently $1 - F(p_\rho) - p_\rho f(p_\rho) = 0$ from Equation (6). The Monotone Hazard Rate assumption ensures the existence and uniqueness of the solution for this problem. We denote this solution as p^+. Consequently, the optimal strategy is characterized as $(p_\rho, A_\rho) = (p^+, 0)$.

Next, one can consider the remaining case with $\frac{1-\tau}{1-r}\frac{\beta(\rho)}{\delta(\rho)} > 1$. A pair of (p_ρ, A_ρ) that satisfies $\frac{\partial \pi_\rho}{\partial p_\rho} = 0$ yields $\frac{\partial \pi_\rho}{\partial A_\rho} > 0$. Such a pair can never be optimal because a marginal increase in A_ρ increases the profit. In contrast, a pair of (p_ρ, A_ρ) that satisfies $\frac{\partial \pi_\rho}{\partial A_\rho} = 0$ yields $\frac{\partial \pi_\rho}{\partial p_\rho} < 0$. Such a pair can be optimal if $p_\rho = 0$ because p_ρ is assumed to take a non-negative value. Given $p_\rho = 0$, the optimal choice of A_ρ is to solve $\frac{\partial \pi_\rho(0, A_\rho)}{\partial A_\rho} = 0$, or equivalently $1 - F(\delta(\rho)A_\rho) = \delta(\rho)A_\rho \cdot f(\delta(\rho)A_\rho) = 0$ from Equation (7). From the fact that $1 - F(x) - x \cdot f(x) = 0$ has a unique solution, it follows that $p^+ = \delta(\rho)A_\rho$. Therefore, the optimal level of A_ρ is equal to $p^+/\delta(\rho)$. Consequently, the optimal strategy is characterized as $(p_\rho, A_\rho) = (0, p^+/\delta(\rho))$. ■

Proof of Corollary 1 Differentiating $(1-\tau)\beta(\hat{\rho}) = (1-r)\delta(\rho)$ with respect to r gives

$$(1-\tau)\beta'(\hat{\rho})\frac{\partial\hat{\rho}(r,\tau)}{\partial r} = -\delta(\hat{\rho}) + (1-r)\delta'(\hat{\rho})\frac{\partial\hat{\rho}(r,\tau)}{\partial r}, \tag{A.1}$$

$$\iff \frac{\partial\hat{\rho}(r,\tau)}{\partial r} = -\frac{\delta(\hat{\rho})}{(1-\tau)\beta'(\hat{\rho}) - (1-r)\delta'(\hat{\rho})} < 0. \tag{A.2}$$

Similarly, differentiating $(1-\tau)\beta(\hat{\rho}) = (1-r)\delta(\rho)$ with respect to τ gives

$$-\beta(\hat{\rho}) + (1-\tau)\beta'(\hat{\rho})\frac{\partial\hat{\rho}(r,\tau)}{\partial \tau} = (1-r)\delta'(\hat{\rho})\frac{\partial\hat{\rho}(r,\tau)}{\partial \tau}, \tag{A.3}$$

$$\iff \frac{\partial\hat{\rho}(r,\tau)}{\partial \tau} = \frac{\beta(\hat{\rho})}{(1-\tau)\beta'(\hat{\rho}) - (1-r)\delta'(\hat{\rho})} > 0. \tag{A.4}$$

■

Proof of Proposition 2 The derivatives of social welfare with respect to r and τ are given as shown below.

$$\frac{\partial W(r,\tau)}{\partial r} = p^+D^+ \cdot \left(-\frac{\beta(\hat{\rho}) - \delta(\hat{\rho})}{\delta(\hat{\rho})} \cdot g(\hat{\rho})\right) \cdot \frac{\partial\hat{\rho}}{\partial r} \tag{A.5}$$

$$\frac{\partial W(r,\tau)}{\partial \tau} = p^+D^+ \cdot \left(-\frac{\beta(\hat{\rho}) - \delta(\hat{\rho})}{\delta(\hat{\rho})} \cdot g(\hat{\rho})\right) \cdot \frac{\partial\hat{\rho}}{\partial \tau} \tag{A.6}$$

The two first-order conditions, $\frac{\partial W(r,\tau)}{\partial r} = 0$ and $\frac{\partial W(r,\tau)}{\partial \tau} = 0$, are redundant. They can be reduced to $\beta(\hat{\rho}(r,\tau)) = \delta(\hat{\rho}(r,\tau))$. Multiple pairs of (r,τ) exist which satisfy the equation. By definition of $\hat{\rho}(r,\tau)$, when $\beta(\hat{\rho}(r,\tau)) = \delta(\hat{\rho}(r,\tau))$ holds, one can obtain the following.

$$\frac{(1-\tau)\beta(\hat{\rho}(r,\tau))}{1-r} = \delta(\hat{\rho}(r,\tau)) \iff \frac{1-\tau}{1-r} = 1 \tag{A.7}$$

$$\iff r = \tau \tag{A.8}$$

■

Proof of Proposition 3 First, we show that assuming Condition (13) ensures that the second-order conditions are satisfied and that r and τ are strategic complements.

Differentiating the first-order condition (11) with respect to τ yields

$$r'(\tau) \cdot \frac{\partial^2 \Pi_{App}}{\partial r^2} + \frac{\partial^2 \Pi_{App}}{\partial r \partial \tau} = 0, \tag{A.9}$$

where

$$\frac{\partial^2 \Pi_{App}}{\partial r^2} = p^+ D^+ \left(2g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} + r g'(\hat{\rho}) \left(\frac{\partial \hat{\rho}}{\partial r} \right)^2 + r g(\hat{\rho}) \frac{\partial^2 \hat{\rho}}{\partial r^2} \right), \tag{A.10}$$

$$\frac{\partial^2 \Pi_{App}}{\partial r \partial \tau} = p^+ D^+ \left(g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial \tau} + r g'(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} \frac{\partial \hat{\rho}}{\partial \tau} + r g(\hat{\rho}) \frac{\partial^2 \hat{\rho}}{\partial r \partial \tau} \right), \tag{A.11}$$

with

$$\frac{\partial^2 \hat{\rho}}{\partial r^2} = \left(\frac{\partial \hat{\rho}}{\partial r} \right)^2 \cdot \left(2 \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \frac{(1-\tau)\beta''(\hat{\rho}) - (1-r)\delta''(\hat{\rho})}{(1-\tau)\beta'(\hat{\rho}) - (1-r)\delta'(\hat{\rho})} \right), \tag{A.12}$$

$$\frac{\partial^2 \hat{\rho}}{\partial \tau^2} = \left(\frac{\partial \hat{\rho}}{\partial \tau} \right)^2 \cdot \left(2 \frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} - \frac{(1-\tau)\beta''(\hat{\rho}) - (1-r)\delta''(\hat{\rho})}{(1-\tau)\beta'(\hat{\rho}) - (1-r)\delta'(\hat{\rho})} \right), \tag{A.13}$$

$$\frac{\partial^2 \hat{\rho}}{\partial r \partial \tau} = \frac{\partial \hat{\rho}}{\partial r} \cdot \frac{\partial \hat{\rho}}{\partial \tau} \left(\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \frac{(1-\tau)\beta''(\hat{\rho}) - (1-r)\delta''(\hat{\rho})}{(1-\tau)\beta'(\hat{\rho}) - (1-r)\delta'(\hat{\rho})} \right). \tag{A.14}$$

Let $\Phi \equiv \frac{(1-\tau)\beta''(\hat{\rho}) - (1-r)\delta''(\hat{\rho})}{(1-\tau)\beta'(\hat{\rho}) - (1-r)\delta'(\hat{\rho})}$. It follows that

$$\frac{\partial^2 \Pi_{App}}{\partial r^2} \cdot \frac{1}{p^+ D^+} \cdot \left(-\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})} \right) \tag{A.15}$$

$$= 2g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial \tau} + r g'(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} \frac{\partial \hat{\rho}}{\partial \tau} + r g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} \frac{\partial \hat{\rho}}{\partial \tau} \left(2 \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \Phi \right) \tag{A.16}$$

$$> g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial \tau} + r g'(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} \frac{\partial \hat{\rho}}{\partial \tau} + r g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} \frac{\partial \hat{\rho}}{\partial \tau} \left(\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \Phi \right) \tag{A.17}$$

$$= \frac{\partial^2 \Pi_{App}}{\partial r \partial \tau} \cdot \frac{1}{p^+ D^+}, \tag{A.18}$$

implying that

$$r'(\tau) = -\frac{\partial^2 \Pi_{App}}{\partial r \partial \tau} \bigg/ \frac{\partial^2 \Pi_{App}}{\partial r^2} < \frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}. \tag{A.19}$$

Therefore, for $\frac{\partial^2 \Pi_{App}}{\partial r \partial \tau} > 0$ and $\frac{\partial^2 \Pi_{App}}{\partial r^2} < 0$, it is sufficient to assume that the right-hand side of inequality (A.17) is positive, or equivalently

$$g(\hat{\rho})\frac{\partial \hat{\rho}}{\partial \tau} + rg'(\hat{\rho})\frac{\partial \hat{\rho}}{\partial r}\frac{\partial \hat{\rho}}{\partial \tau} + rg(\hat{\rho})\frac{\partial \hat{\rho}}{\partial r}\frac{\partial \hat{\rho}}{\partial \tau}\left(\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \Phi\right) > 0, \tag{A.20}$$

$$\iff 1 + r\frac{g'(\hat{\rho})}{g(\hat{\rho})}\frac{\partial \hat{\rho}}{\partial r} + r\frac{\partial \hat{\rho}}{\partial r}\left(\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \Phi\right) > 0, \tag{A.21}$$

$$\iff 1 - r\frac{\partial \hat{\rho}}{\partial r}\left(-\frac{g'(\hat{\rho})}{g(\hat{\rho})} - \frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} - \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} + \Phi\right) > 0. \tag{A.22}$$

Consequently, $\Phi > \frac{g'(\hat{\rho})}{g(\hat{\rho})} + \frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})}$ is a sufficient condition for inequality (A.22) to hold, which is a part of Condition (13).

Similarly, differentiating the first-order condition (12) with respect to r yields

$$\tau'(r) \cdot \frac{\partial^2 \Pi_{Ad}}{\partial \tau^2} + \frac{\partial^2 \Pi_{Ad}}{\partial r \partial \tau} = 0, \tag{A.23}$$

where

$$\frac{\partial^2 \Pi_{Ad}}{\partial \tau^2} = p^+ D^+ \left(-2\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial \hat{\rho}}{\partial \tau} - \tau\left(\frac{\beta}{\delta}g\right)'\left(\frac{\partial \hat{\rho}}{\partial \tau}\right)^2 - \tau\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial^2 \hat{\rho}}{\partial \tau^2}\right), \tag{A.24}$$

$$\frac{\partial^2 \Pi_{Ad}}{\partial r \partial \tau} = p^+ D^+ \left(-\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial \hat{\rho}}{\partial r} - \tau\left(\frac{\beta}{\delta}g\right)'\frac{\partial \hat{\rho}}{\partial r}\frac{\partial \hat{\rho}}{\partial \tau} - \tau\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial^2 \hat{\rho}}{\partial r \partial \tau}\right). \tag{A.25}$$

It follows that

$$\frac{\partial^2 \Pi_{Ad}}{\partial \tau^2} \cdot \frac{1}{p^+ D^+} \cdot \left(-\frac{\delta(\hat{\rho})}{\beta(\hat{\rho})}\right) \tag{A.26}$$

$$= -2\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial \hat{\rho}}{\partial r} - \tau\left(\frac{\beta}{\delta}g\right)'\frac{\partial \hat{\rho}}{\partial r}\frac{\partial \hat{\rho}}{\partial \tau} - \tau\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial \hat{\rho}}{\partial r}\frac{\partial \hat{\rho}}{\partial \tau}\left(2\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} - \Phi\right) \tag{A.27}$$

$$> -\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial \hat{\rho}}{\partial r} - \tau\left(\frac{\beta}{\delta}g\right)'\frac{\partial \hat{\rho}}{\partial r}\frac{\partial \hat{\rho}}{\partial \tau} - \tau\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})}g(\hat{\rho})\frac{\partial \hat{\rho}}{\partial r}\frac{\partial \hat{\rho}}{\partial \tau}\left(\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \Phi\right) \tag{A.28}$$

$$= \frac{\partial^2 \Pi_{Ad}}{\partial r \partial \tau} \cdot \frac{1}{p^+ D^+}, \tag{A.29}$$

implying that

$$\tau'(r) = -\frac{\partial^2 \Pi_{Ad}}{\partial r \partial \tau} \Big/ \frac{\partial^2 \Pi_{Ad}}{\partial \tau^2} < \frac{\delta(\hat{\rho})}{\beta(\hat{\rho})}. \tag{A.30}$$

Therefore, for $\frac{\partial^2 \Pi_{Ad}}{\partial r \partial \tau} > 0$ and $\frac{\partial^2 \Pi_{Ad}}{\partial \tau^2} < 0$, it is sufficient to assume that the right-hand side of inequality (A.28) is positive, or equivalently

$$-\frac{\beta(\hat{\rho})}{\delta(\hat{\rho})} g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} - \tau \left(\frac{\beta}{\delta} g \right)' \frac{\partial \hat{\rho}}{\partial r} \frac{\partial \hat{\rho}}{\partial \tau} - \tau \frac{\beta(\hat{\rho})}{\delta(\hat{\rho})} g(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial r} \frac{\partial \hat{\rho}}{\partial \tau} \left(\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} + \frac{\delta'(\hat{\rho})}{\delta(\hat{\rho})} - \Phi \right) > 0 \tag{A.31}$$

$$\Longleftrightarrow \ 1 + \tau \frac{\partial \hat{\rho}}{\partial \tau} \left(\frac{g'(\hat{\rho})}{g(\hat{\rho})} + 2 \frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})} - \Phi \right) > 0 \tag{A.32}$$

Consequently, $\Phi < \frac{g'(\hat{\rho})}{g(\hat{\rho})} + 2\frac{\beta'(\hat{\rho})}{\beta(\hat{\rho})}$ is a sufficient condition for inequality (A.32) to hold, which is a part of Condition (13).

In total, Condition (13) ensures that strategic variables r and τ are strategic complements (i.e., $r'(\tau) > 0$ and $\tau'(r) > 0$). Consequently, there exists a unique equilibrium of (r^*, τ^*) that satisfies the first-order conditions (11) and (12). Moreover, from inequalities (A.19) and (A.30), it follows that $r'(\tau) \cdot \tau'(r) < 1$, implying that the unique equilibrium is stable. Furthermore, the second-order conditions are satisfied.

Next, we obtain the necessary and sufficient condition for which paid apps are oversupplied in terms of social welfare (i.e., Condition (14)).

From the app platform's first-order condition (11), the equilibrium (r^*, τ^*) satisfies

$$G(\rho^*) + r g(\rho^*) \frac{\partial \hat{\rho}(r^*, \tau^*)}{\partial r} = 0, \tag{A.33}$$

or equivalently

$$\frac{G(\rho^*)}{g(\rho^*)} \{(1 - \tau^*)\beta'(\rho^*) - (1 - r^*)\delta(\rho^*)\} - r^* \delta(\rho^*) = 0. \tag{A.34}$$

Similarly, from the ad platform's first-order condition (12), the equilibrium

(r^*, τ^*) satisfies

$$\int_{\rho^*}^{\overline{\rho}} \frac{\beta(\rho)}{\delta(\rho)} dG(\rho) - \tau^* \frac{\beta(\rho^*)}{\delta(\rho^*)} g(\rho^*) \frac{\partial \hat{\rho}(r^*, \tau^*)}{\partial \tau} = 0, \tag{A.35}$$

or equivalently

$$\frac{\delta(\rho^*)}{\beta(\rho^*)} \{(1-\tau^*)\beta'(\rho^*) - (1-r^*)\delta(\rho^*)\} \frac{1}{g(\rho^*)} \int_{\rho^*}^{\overline{\rho}} \frac{\beta(\rho)}{\delta(\rho)} dG(\rho) - \tau^* \beta(\rho^*) = 0. \tag{A.36}$$

Subtracting Equation (A.36) from Equation (A.34) yields

$$\frac{(1-\tau^*)\beta'(\rho^*) - (1-r^*)\delta(\rho^*)}{g(\rho^*)} \cdot \left(G(\rho^*) - \frac{\delta(\rho^*)}{\beta(\rho^*)} \int_{\rho^*}^{\overline{\rho}} \frac{\beta(\rho)}{\delta(\rho)} dG(\rho) \right) - r^* \delta(\rho^*) + \tau^* \beta(\rho^*) = 0. \tag{A.37}$$

Here, by definition of $\hat{\rho}$, it follows that $(1-\tau^*)\beta(\rho^*) = (1-r^*)\delta(\rho^*)$, or equivalently $\beta(\rho^*) - \delta(\rho^*) = -r^*\delta(\rho^*) + \tau^*\beta(\rho^*)$. Therefore, Equation (A.37) can be rewritten as follows.

$$\beta(\rho^*) - \delta(\rho^*) = \frac{(1-\tau^*)\beta'(\rho^*) - (1-r^*)\delta(\rho^*)}{g(\rho^*)} \cdot \left(\frac{\delta(\rho^*)}{\beta(\rho^*)} \int_{\rho^*}^{\overline{\rho}} \frac{\beta(\rho)}{\delta(\rho)} dG(\rho) - G(\rho^*) \right) \tag{A.38}$$

Therefore, $\beta(\rho^*) > \delta(\rho^*)$ holds if and only if Condition (14) holds. ■

Proof of Corollary 2 First, because $\frac{\beta(\rho)}{\delta(\rho)}$ is an increasing function, the left-hand side of Condition (14) is at least greater than $1 - G(\rho^*)$, as shown below.

$$\frac{\delta(\rho^*)}{\beta(\rho^*)} \int_{\rho^*}^{\overline{\rho}} \frac{\beta(\rho)}{\delta(\rho)} dG(\rho) > \frac{\delta(\rho^*)}{\beta(\rho^*)} \cdot \frac{\beta(\rho^*)}{\delta(\rho^*)} \int_{\rho^*}^{\overline{\rho}} dG(\rho) = 1 - G(\rho^*) \tag{A.39}$$

Consequently, for paid apps being oversupplied, it is sufficient to satisfy $1 - G(\rho^*) > G(\rho^*)$, or equivalently $G(\rho^*) < 1/2$.

Proof of Proposition 6 Because of $\tau^{**} = 0$, it holds that $\Pi^1_{Ad}(r^{**}, \tau^{**}) = 0$. Consequently, we show hereinafter that $\Pi_{App}(r^*, \tau^*) > \Pi_{App}(r^{**}, 0)$. First, it follows that

$$\frac{\partial \Pi_{App}}{\partial \tau} = rp^{+}D^{+} \cdot G(\hat{\rho}) \cdot \frac{\partial \hat{\rho}}{\partial \tau} > 0, \tag{A.40}$$

implying that $\Pi_{App}(r^{**}, 0) < \Pi_{App}(r^{**}, \tau^{*})$ holds. Next, by definition, it follows that $\Pi_{App}(r^{**}, \tau^{*}) < \Pi_{App}(r^{*}, \tau^{*})$. Considered comprehensively, $\Pi_{App}(r^{*}, \tau^{*}) > \Pi_{App}(r^{**}, 0)$ holds. ■

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